Table of Contents

Acknowledgements **v**

Abstract **ix**

Deutsche Kurzfassung **xi**

1 Introduction **1**
1.1 Motivation and Background 1
1.2 Problem Statement 3
1.3 Contributions and Outline 5

2 Related Work and Main Idea **7**
2.1 Related Work and Literature Review 7
2.2 Extremum Seeking 11
2.3 Main Idea of Gradient Approximation 13

3 Derivative-Free Optimization based on Non-Commutative Maps **19**
3.1 The Algorithmic Scheme 19
3.2 Convergence 22
3.3 Exploration Sequence 23
3.4 Gradient-Generating Functions 28
3.5 Algorithm Design 30
3.6 Conclusion and Outlook 36

4 Numerical Results and Experiments **39**
4.1 Algorithm Implementation and Setup 40
4.2 Design Parameters and Functions: Qualitative Study 40
4.3 Design Parameters and Functions: Quantitative Study 50
4.4 Performance Tuning 53
4.5 Benchmarking 60
4.6 Direct Policy Search for Reinforcement Learning 64
4.7 Conclusion and Outlook 68

5 Summary and Conclusion **69**

A Notation, Background, and Preliminary Lemmas **73**
A.1 Notation 73
A.2 Technical Background 74
A.3 Preliminary Lemmas and Results 76

B Proofs **83**
B.1 Proof of Theorem 1 . . . 83
B.2 Proof of Theorem 2 . . . 83
B.3 Proof of Theorem 3 . . . 85
B.4 Proof of Lemma 1 . . . 85
B.5 Proof of Theorem 4 . . . 86
B.6 Proof of Lemma 2 . . . 88
B.7 Proof of Theorem 5 . . . 90
B.8 Proof of Theorem 6 . . . 93

C Additional Material on Numerical Results **95**
C.1 Parameter Study . . . 95
C.2 Benchmarking . . . 106

D Construction of Exploration Sequence Matrix W **115**
D.1 Step-by-step Construction of W . . . 115
D.2 Construction Procedure of a Principal Submatrix . . . 117

List of Symbols **123**

Bibliography **127**