

Contents

| | |
|---|------------|
| Acknowledgement | iii |
| 1 Introduction | 1 |
| 1.1 Motivation | 2 |
| 1.2 Overview of the relevant regularity theory | 6 |
| 1.3 The thesis in a nutshell | 9 |
| 1.4 Outline | 13 |
| 2 Preliminaries | 15 |
| 2.1 Some conventions | 15 |
| 2.1.1 Bounded operators | 15 |
| 2.1.2 Domains in \mathbb{R}^d | 16 |
| 2.1.3 Measurable mappings and L_p -spaces | 17 |
| 2.1.4 Probabilistic setting | 18 |
| 2.1.5 Functions, distributions and the Fourier transform | 20 |
| 2.1.6 Miscellaneous notation | 22 |
| 2.2 Stochastic integration in UMD Banach spaces | 22 |
| 2.2.1 Geometric properties of Banach spaces | 23 |
| 2.2.2 γ -radonifying operators | 25 |
| 2.2.3 Stochastic integration for cylindrical Brownian motions | 27 |
| 2.3 Function spaces | 31 |
| 2.3.1 Sobolev spaces | 31 |
| 2.3.2 Spaces of Bessel potentials | 33 |
| 2.3.3 Weighted Sobolev spaces | 37 |
| 2.3.4 Besov spaces | 46 |
| 2.3.5 Triebel-Lizorkin spaces | 50 |
| 2.4 Semigroups of linear operators | 51 |
| 3 Starting point: Linear SPDEs in weighted Sobolev spaces | 55 |
| 3.1 Stochastic parabolic weighted Sobolev spaces $\mathfrak{H}_{p,\theta}^{\gamma,q}(G, T)$ | 57 |
| 3.2 An L_p -theory of linear SPDEs on bounded Lipschitz domains | 61 |
| 4 Embeddings of weighted Sobolev spaces into Besov spaces | 67 |
| 4.1 Weighted Sobolev spaces and Sobolev spaces without weights | 68 |
| 4.2 Wavelet decomposition of Besov spaces on \mathbb{R}^d | 70 |
| 4.3 Weighted Sobolev spaces and the non-linear approximation scale | 71 |
| 4.4 An alternative proof of Theorem 4.7 | 79 |

| | | |
|----------|---|------------|
| 5 | Spatial Besov regularity of SPDEs on bounded Lipschitz domains | 85 |
| 5.1 | Linear equations | 86 |
| 5.2 | Semi-linear equations | 95 |
| 6 | Space time regularity of the inhomogeneous heat equation with additive noise | 107 |
| 6.1 | Space time regularity of elements from $\mathfrak{H}_{p,\theta}^{\gamma,q}(\mathcal{O}, T)$ | 109 |
| 6.2 | The spaces $\mathfrak{H}_{p,\theta}^{\gamma,q}(\mathcal{O}, T)$ and SPDEs | 117 |
| 6.3 | The stochastic heat equation in $\mathfrak{H}_{p,\theta}^{\gamma,q}(\mathcal{O}, T)$ | 121 |
| 6.3.1 | A result on the $L_q(L_p)$ -regularity | 122 |
| 6.3.2 | Space time regularity | 127 |
| | Zusammenfassung | 131 |
| | Notation | 137 |
| | Bibliography | 143 |
| | Index | 151 |