Abstract

We present a framework for data-driven model predictive control (MPC) with theoretical guarantees on closed-loop stability and robustness. The proposed approach relies on Willems' Fundamental Lemma which parametrizes all trajectories of an unknown linear system based on one measured input-output trajectory. This result can be used to design MPC schemes simply from measured trajectories of the system rather than from its model, which need not be known. However, when applying such a scheme in closed loop, stability is not necessarily guaranteed. To close this gap, we develop a framework for designing and analyzing MPC schemes, which are only based on measured data and do not require explicit model knowledge, and come with rigorous closed-loop guarantees. To this end, we consider three distinct problem setups of increasing difficulty: data-driven MPC for linear systems with noise-free data, for linear systems with noisy data, and for nonlinear systems, respectively.

Data-driven MPC for linear systems with noise-free data

Chapter 3 addresses MPC for linear time-invariant (LTI) systems based on one noise-free input-output trajectory. More specifically, three data-driven MPC schemes are investigated for this scenario. First, we propose to use terminal equality constraints which enforce the input and output to be equal to the setpoint for several time steps at the end of the prediction horizon. Next, we develop a data-driven MPC scheme that contains general terminal ingredients, i.e., a terminal cost and a terminal constraint, and we provide a procedure to design such terminal ingredients based on input-output data. Finally, we suggest a tracking MPC formulation containing an artificial setpoint which is optimized online. For each of these schemes, we prove recursive feasibility, satisfaction of input and output constraints, as well as stability of the closed loop.

Data-driven MPC for linear systems with noisy data

In Chapter 4, we consider MPC for LTI systems based on one input-output trajectory which is affected by measurement noise. We prove that, by adding a slack variable and suitable regularization terms, the data-driven MPC with terminal equality constraints practically exponentially stabilizes the closed loop. Additionally, we derive a constraint tightening to ensure that the output robustly satisfies the constraints. Finally, we present a robust data-driven tracking MPC scheme which combines robustifying ingredients such as a slack variable and regularization with an artificial equilibrium. Based on a novel separation argument of nominal and robust data-driven MPC, we prove that this scheme practically exponentially stabilizes the optimal reachable equilibrium for the given setpoint.

Data-driven MPC for nonlinear systems

In Chapter 5, we address MPC for unknown nonlinear systems based only on input-output data. In contrast to the approaches for linear systems, we update the data online at each step, thereby implicitly exploiting local linear approximations of the underlying nonlinear dynamics. We prove that, if a certain cost parameter is sufficiently small and the initial state is close to the steady-state manifold, then the proposed data-driven MPC scheme practically exponentially stabilizes the optimal reachable equilibrium for the unknown nonlinear system. As an intermediate result of independent interest, we also develop and analyze a model-based MPC scheme to control nonlinear systems by using the linearized dynamics at the current state as the prediction model.

In summary, the main goal of this thesis is to develop a framework for MPC based only on measured data with stability and robustness guarantees for the closed loop. To this end, we merge existing model-based MPC results with Willems' Fundamental Lemma, and we provide a detailed analysis of the effect of noise or nonlinear dynamics on the closed-loop behavior. Moreover, we demonstrate with numerical as well as experimental examples that the proposed data-driven MPC framework not only admits strong theoretical guarantees, but is also simple to apply and provides high performance for challenging control problems.

Deutsche Kurzfassung

Diese Doktorarbeit befasst sich mit der Entwicklung und Analyse von Verfahren zur prädiktiven Regelung (Englisch: *model predictive control*, MPC) basierend auf gemessenen Daten mit theoretischen Garantien für Stabilität und Robustheit. Der vorgestellte Ansatz beruht auf dem *Fundamental Lemma* von Willems et al., welches sämtliche Trajektorien eines unbekannten linearen Systems basierend auf einer gemessenen Eingangs-Ausgangs-Trajektorie parametrisiert. Mit Hilfe dieses Resultats können MPC-Algorithmen entworfen werden, indem das übliche Zustandsraummodell durch datenabhängige Hankel-Matrizen ersetzt wird. Im Allgemeinen ist nicht garantiert, dass der aus der Anwendung dieses Schemas resultierende geschlossene Kreis stabil ist. Zur Lösung dieses Problems entwickeln wir verschiedene Ansätze zum Entwurf und zur Analyse von MPC-Algorithmen, welche ausschließlich auf gemessenen Daten basieren und kein explizites Modellwissen benötigen, jedoch rigorose Garantien für den geschlossenen Regelkreis aufweisen. Wir betrachten drei verschiedene Problemstellungen mit zunehmendem Schwierigkeitsgrad: Datenbasierte MPC-Methoden für lineare Systeme mit rauschfreien Daten, für lineare Systeme mit verrauschten Daten und für nichtlineare Systeme.

Datenbasierte MPC-Methoden für lineare Systeme mit rauschfreien Daten

In Kapitel 3 behandeln wir MPC für lineare, zeitinvariante Systeme basierend auf einer rauschfreien Eingangs-Ausgangs-Trajektorie. Im Speziellen entwickeln wir drei MPC-Algorithmen für dieses Szenario. Zunächst verwenden wir Gleichheitsbeschränkungen, welche erzwingen, dass die prädizierten Signale gegen Ende des Prädiktionshorizonts gleich dem Sollwert sind. Anschließend stellen wir einen datenbasierten MPC-Algorithmus vor, welcher allgemeine Endbedingungen enthält, d.h. eine Endkostenfunktion sowie eine Zielmenge. Ferner entwickeln wir eine Methode zur Auslegung solcher Endbedingungen basierend auf Eingangs-Ausgangs-Daten. Schließlich präsentieren wir eine MPC-Formulierung basierend auf einer

künstlichen Referenz, welche während des Betriebs optimiert wird. Für jeden der Algorithmen beweisen wir rekursive Lösbarkeit, die Erfüllung von Eingangs- und Ausgangs-Beschränkungen sowie Stabilität des geschlossenen Kreises.

Datenbasierte MPC-Methoden für lineare Systeme mit verrauschten Daten

Kapitel 4 beschreibt MPC-Methoden für lineare, zeitinvariante Systeme basierend auf einer Eingangs-Ausgangs-Trajektorie, welche von Messrauschen beeinflusst ist. Wir beweisen, dass der datenbasierte MPC-Algorithmus mit Gleichheitsbeschränkungen durch Hinzufügen einer Schlupfvariable sowie geeigneter Regularisierungsterme zu praktischer exponentieller Stabilität führt. Ebenso leiten wir striktere Beschränkungen her, welche sicherstellen, dass der Ausgang die originalen Beschränkungen trotz der verrauschten Daten robust einhält. Schließlich entwickeln wir einen robusten datenbasierten MPC-Algorithmus, welcher robustheitserzeugende Komponenten (Schlupfvariable und Regularisierung) mit einer künstlichen Referenz verknüpft. Mit Hilfe eines neuartigen Separationsarguments für nominelle und robuste datenbasierte MPC-Algorithmen beweisen wir, dass dieser Algorithmus das bestmögliche Gleichgewicht für die gegebene Referenz praktisch exponentiell stabilisiert.

Datenbasierte MPC-Methoden für nichtlineare Systeme

Kapitel 5 adressiert MPC-Methoden für unbekannte nichtlineare Systeme basierend auf Eingangs-Ausgangs-Daten. Im Gegensatz zu den Ansätzen für lineare Systeme werden die Daten in jedem Zeitschritt durch neue Messungen ersetzt, wodurch implizit lokale lineare Approximationen des zugrundeliegenden nichtlinearen Systems ausgenutzt werden. Unter den Annahmen, dass eine bestimmte Kostenmatrix klein genug gewählt wird und der Anfangszustand sich nahe an der Gleichgewichtsmannigfaltigkeit befindet, beweisen wir, dass der vorgestellte MPC-Algorithmus das bestmögliche Gleichgewicht für das unbekannte nichtlineare System praktisch exponentiell stabilisiert. Als Zwischenresultat von unabhängigem Interesse entwickeln und analysieren wir einen modellbasierten MPC-Algorithmus für nichtlineare Systeme, welcher zur Prädiktion nur die linearisierte Systemdynamik um den aktuellen Zustand verwendet.

Das Hauptziel dieser Doktorarbeit ist die Entwicklung von Entwurfs- und Analyseverfahren für MPC-Algorithmen basierend auf gemessenen Daten mit Stabilitäts- und Robustheitsgarantien. Hierzu verbinden wir existierende modellbasierte MPC-Resultate mit dem *Fundamental Lemma* von Willems et al. und wir analysieren den Einfluss von Messrauschen oder Nichtlinearitäten auf das geregelte System. Außerdem demonstrieren wir anhand von numerischen sowie experimentellen Ergebnissen, dass der vorgestellte datenbasierte MPC-Ansatz nicht nur wünschenswerte theoretische Garantien aufweist, sondern einfach anzuwenden ist und eine hohe Regelgüte für anspruchsvolle praktische Regelungsprobleme aufweist.

Chapter 1

Introduction

1.1 Motivation

Classical controller design requires model knowledge of the plant. For example, if a controller is to be designed based on methods from nonlinear [67], robust [160], or model predictive [118] control, then typically a state-space model of the underlying plant must be available. However, determining an accurate model can be cumbersome and is in many cases the most time-consuming task in the controller design. On the other hand, it is often easily possible and cheap to gather data by exciting the system in an experiment. With this motivation, the field of Reinforcement Learning [130] has received huge interest in recent years and often exhibits remarkable performance in practical applications. Yet, despite their empirical success, many of these approaches do not admit strong theoretical guarantees such that methods can behave unpredictably or fail, even in very simple applications, see, e.g., [120].

Therefore, there has been an increasing effort in employing control-theoretic tools to use data for control purposes. Arguably, the most intuitive approach to design controllers based on data consists of a two-step procedure: 1) estimating a model from the available data and 2) using the identified model to design a controller via model-based techniques. The field of system identification, which deals with the estimation of models from data, is well-established and contains a wide range of different approaches, see, e.g., [85]. When aiming at theoretical guarantees for a controller designed via an identified model, then two ingredients are required: 1) a model estimate and 2) a bound on the estimation error. In particular, if the model does not exactly represent the underlying system and no bound on the model error is known, then it cannot be guaranteed that the resulting controller will satisfy control goals, e.g., stabilize the underlying system. However, determining the above two ingredients based on data is challenging and an active field of research even for linear time-invariant (LTI) systems, in particular in the inevitable scenario where the data are affected by noise, see, e.g., [32, 100, 101] for stochastic noise. If the noise admits a deterministic description, set membership estimation can be used to obtain a model, where obtaining computationally tractable models with tight error bounds is a key challenge [12, 106]. For nonlinear systems, obtaining accurate models with corresponding error bounds is even more involved: Only few approaches admit strong guarantees and they often rely on Lipschitz continuity-like properties [21, 105], which may lead to complex models, or they require appropriate choices of basis functions [128].

As an alternative to the above *indirect* approach, consisting of sequential system identification and model-based controller design, there have been many works on *direct* data-driven control which avoid the intermediate identification step. This includes techniques such as virtual reference feedback tuning [22], iterative feedback tuning [58], or unfalsification-based approaches [73], see the survey [59] for details and further approaches. As for the indirect approach, however, providing rigorous guarantees based on a finite set of noisy data points is challenging, not only for nonlinear systems but also in the LTI case. To summarize, while the established literature contains different indirect and direct approaches for using data in control, typically no strong theoretical guarantees can be given, in particular in the presence of noise or for nonlinear systems.

The *Fundamental Lemma* proposed by Willems et al. [144] has shown to be a promising basis for data-driven control. The result allows to directly parametrize all trajectories of an unknown LTI system based on one input-output trajectory. Although the paper [144] was published in 2005, the Fundamental Lemma has only received increasing attention in recent years, with many different applications ranging from signal processing and system analysis to controller design, as surveyed in [92]. One of the most prominent and powerful applications of the Fundamental Lemma is the design of data-driven model predictive control (MPC) approaches, as first recognized by [27, 150]. Based on these early works, different extensions and modifications have been developed to cope with noisy data and nonlinear systems, e.g., via regularization [92, Section 5.2.2]. These approaches not only admit rigorous guarantees but also have remarkable performance when used in challenging realworld applications [92, Section 5.2.4]. However, the mentioned theoretical results mainly focus on certifying robustness and performance of the underlying *open-loop* optimal control problem. On the contrary, if we want to implement a feedback controller, then these optimal control problems need to be solved repeatedly in a receding-horizon fashion as in standard (model-based) MPC [118]. For this scenario, the above approaches provide no theoretical guarantees, e.g., on stability or robustness, neither for linear nor nonlinear systems and neither for noise-free nor noisy data.

This motivates the purpose of the present thesis: We develop a framework for designing and analyzing data-driven MPC schemes based on the Fundamental Lemma using only measured input-output data and no explicit model knowledge. Motivated by the above discussion, we derive theoretical guarantees on *closed-loop* stability and robustness, not only for the case of linear systems with noise-free data, but also for the challenging scenarios of noisy data and nonlinear systems. In the following, we discuss literature related to this thesis in more detail, followed by a description of the contributions and outline of the thesis.

1.2 Related work

In this section, we briefly survey selected topics on MPC and data-driven control which are relevant for this thesis. Further details on the relation of our results to existing works will be discussed in the respective sections later in the thesis.

Model predictive control

MPC is a well-established control technique which can handle hard constraints, performance criteria, and nonlinear multi-input multi-output systems. It is based on repeatedly solving an open-loop optimal control problem at each time instant and only applying the first part of the optimal input trajectory in closed loop, see, e.g., [118] for an introduction. To predict future trajectories in this optimal control problem, a *model* of the plant is required. Hence, standard MPC approaches are model-based, although model inaccuracies can also be handled via robust MPC techniques [71, 103, 117]. Further, MPC typically relies on state measurements, whereas output-feedback MPC schemes require an additional state estimation step,