Abstract

In the past decade the use of autonomously moving objects called 'agents' has become increasingly popular. In particular, unmanned aerial vehicles (UAVs) are used for a wide range of tasks due to their low operating and maintenance costs. At the same time, the greater number of UAVs increases the risk of collisions. This thesis proposes a method that plans trajectories for agents to enable them to fulfil individual tasks as well as cooperative tasks in groups by ensuring the collision-free movement. The agents are locally controlled and connected over an unreliable communication network that may induce packet losses and transmission delays. Further sensors e.g. for a distance measurement are not used and communication should only be invoked if it is necessary to avoid a collision.

The basic problem occurs for two agents. The first agent is called the stand-on agent, it can change its trajectory at any time without regard to the second agent. This agent is named the give-way agent. It has to ensure the collision avoidance by adapting its trajectory based on local data and communicated information about the current and future movement of the stand-on agent. A control unit for the give-way agent is introduced that has to execute four tasks to ensure the control aims: 1. Estimation of the current network properties. 2. Prediction of the movement of the stand-on agent. 3. Invocation of communication whenever the local data becomes too uncertain. 4. Planning of collision avoiding trajectories.

The aim of this thesis is to provide methods so as to solve the four tasks. To this aim a control unit consisting of four parts is introduced that uses approaches from control theory and communication technology. A delay estimator generates an estimate of the properties of the communication network that vary with the distance between the agents. A prediction unit determines a set that includes the uncertain future movement of the neighbouring agent. An event generator monitors the control aims and invokes communication in an event-based fashion when the local data becomes too uncertain. Furthermore, it decides when it is necessary to change the trajectory of the agent in order to ensure the collision-free movement. A trajectory planning unit provides collision-free trajectories based on Bézier curves. As a result it is proven that collisions are avoided using the proposed method even in the presence of transmission delays and packet losses induced by the communication network. It is shown that the network estimation is suitable for the controller and the set of predicted positions of the nearby agent is feasible.

The main result of the thesis is a novel control method for mobile agents that are connected over an unreliable communication network. Local trajectories are planned for the agents so that they are able to fulfil individual tasks as well as cooperative tasks in a group with a guaranteed collision avoidance. Communication is only utilised whenever it is necessary. The method can be used for different types of agents (e.g. UAVs or cars) with only slight modifications. The proposed methods are tested and evaluated through simulations and experiments with two quadrotors.

Introduction to the event-based control of mobile agents

1

1.1 Purpose of the cooperative control of mobile agents

The use of autonomously moving objects such as unmanned aerial vehicles (UAVs) or cars enables one the execution of individual tasks, e.g. parcel delivery by using drones. Another application example is illustrated in Fig. 1.1. Here, two UAVs are utilised to provide communication to several moving ground objects, which are not able to communicate directly to each other. Due to the large distances between the ground objects two UAVs are required that act as aerial communication relay stations to provide all objects with communication links.

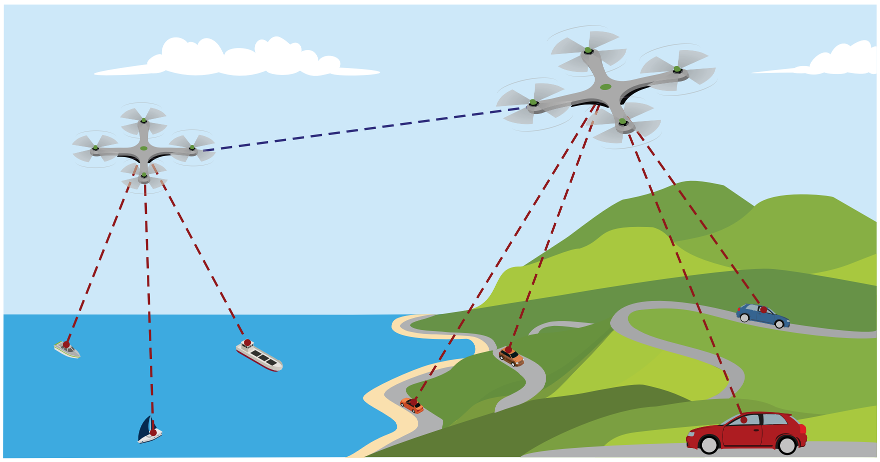

Fig. 1.1: Application example: Communication between ground objects using UAVs as aerial communication relay stations.

This application example illustrates that the autonomous movements of the UAVs need to be coordinated to fulfil their individual tasks. Furthermore, the increasing number of unmanned moving objects raises the risk of collisions. This thesis provides a method for the cooperative control of autonomously moving objects called *agents* in the literature of control theory. The agents are connected over a communication network and should fulfil the following control aims:

1. Collision avoidance between autonomously moving agents.

2. Satisfaction of individual tasks of the agents (e.g. reaching their individual destinations).

As the existing communication networks are increasingly utilised to capacity, with the control method the control aims should be satisfied with a significantly reduced communication exchange between the agents. To this aim the method uses the idea of event-based control from control theory and combines it with models from communication technology.

1.2 Event-based control of mobile agents

The agents to be considered are only able to measure their own positions and speeds locally to save energy. They communicate these data together with their current trajectory to the neighbouring agent. As the agents can change their trajectories at any time (e.g. due to an obstacle on their trajectory), a conflict of the control aims introduced in the preceding section may occur. However, not every change of the trajectories leads directly to a conflict of the control aims. Hence, a continuous communication is not necessary, but it is sufficient to communicate and to change the trajectories of the agents only if the uncertainties with respect to the control aims become too large. For this reason, the idea of event-based control is utilised, where information are only sent when a threshold is violated that indicates the uncertainty of the local information.

The problem statement of this thesis results from a conflict of the control aims. In this case the control method to be developed should plan the trajectories for the agents so as to fulfil the following two objectives:

- Avoid collisions between the agents by maintaining a safety distance between them.
- Maintain a maximum distance between the agents.

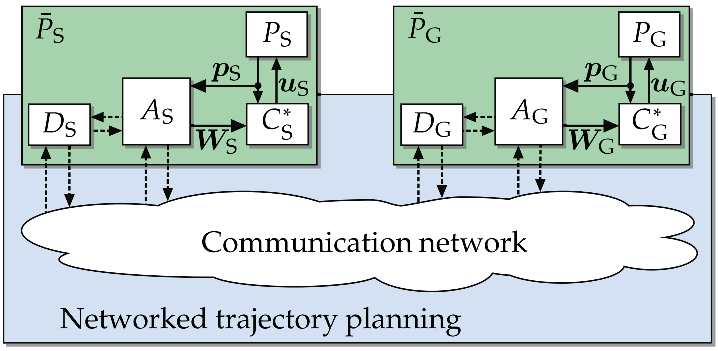

Fig. 1.2: Structure of the networked control system.

In the following, the networked system described in Section 1.1 is transferred into the control engineering structure, which is depicted in Fig. 1.2. In this thesis the control system consists of two mobile agents that are assigned with the functions *stand-on*[1] (agent $\bar{P}_{\mathrm{S}}$) and *give-way*[1] (agent $\bar{P}_{\mathrm{G}}$). The assignment of the tasks to the agents depends on the current situation as illustrated in Section 1.3.2. The stand-on agent $\bar{P}_{\mathrm{S}}$ and the give-way agent $\bar{P}_{\mathrm{G}}$ consist of the physical objects P_{S} and P_{G}, which are independently controlled by local flatness-based two-degrees-of-freedom controllers C^*_{S} and C^*_{G}. They generate the control inputs $\boldsymbol{u}_{\mathrm{S}}(t)$ and $\boldsymbol{u}_{\mathrm{G}}(t)$ and make the objects follow their trajectories generated by the corresponding event-based control units A_{S} and A_{G} even in the presence of external disturbances and model uncertainties. These control units use the locally measured positions $\boldsymbol{p}_{\mathrm{S}}(t)$ and $\boldsymbol{p}_{\mathrm{G}}(t)$ of the agents to generate the matrices $\boldsymbol{W}_{\mathrm{S}}(t)$ and $\boldsymbol{W}_{\mathrm{G}}(t)$, which consist of the trajectories and their first four derivatives and are able to communicate over the communication network if necessary.

The communication network is assumed to be unreliable in the sense that data packets may be received delayed or lost. The delay estimators D_{S} and D_{G} contain a Markov model that provides an estimate of the current properties of the network to take them into account for the trajectory planning.

Remark. The controllers C^*_{S} and C^*_{G} are designed using conventional control engineering methods. The design of the controllers depends on the specific type of the object to be controlled. First, the thesis focusses on the design of the event-based control units A_{S} and A_{G}, the controllers are derived in Chapter 10 for a quadrotor as the demonstration example.

In order to achieve both objectives stated above in a distributed fashion the control units A_{S} and A_{G} of the agents execute the following four tasks in an event-based fashion:

1. Network estimation: Estimate the current quality-of-service (QoS) properties of the communication channel, which vary with the distance between the agents.

2. Prediction: Estimate the future positions of the neighbouring agent.

3. Communication: Invoke communication by incorporating the estimated QoS properties of the channel only when the local data becomes too uncertain indicated by a threshold.

4. Trajectory planning: Plan the trajectory so as to fulfil the control aims. Modify the trajectory whenever a violation of the control aims threatens due to a changed situation.

The aim of this thesis is to elaborate methods to solve the four tasks. Particular attention is laid on the fact that communication between the agents is only invoked if it is absolutely necessary. The collision avoidance should be guaranteed even in the presence of

[1]The term is motivated by the International Regulations for Preventing Collisions at Sea 1972 (COLREGs) for maritime navigation [95].

transmission delays or packet losses induced by the communication network. In addition, the method should be able to control different types of agents (e.g. UAVs, cars) with only small modifications.

1.3 Motivation and application fields

1.3.1 *Motivation for the event-based control of mobile agents*

Practical motivation. Autonomous moving agents especially UAVs are used in many different applications due to their ease of deployment, low maintenance cost, high mobility and ability to hover. Such vehicles are utilised, for example, for providing wireless coverage, real-time monitoring of road traffic, remote sensing or search and rescue operations. Often an UAV is deployed to perform various tasks and some tasks require the cooperation of several UAVs in a formation. Hence, it is appropriate to develop a method with which both, a single agent or a group of agents is able to execute different types of tasks. Furthermore, the increase of UAVs in air traffic raises the risk of collisions. As no pilot is on board, the method to be developed has to ensure the collision-free movement of the agents while fulfilling their tasks.

In order to ensure the collision-free movement and to fulfil the individual tasks of the agents, the positions of nearby agents must be known continuously. In common practice, these positions are measured by using on-board sensors or cameras, which causes the following difficulties:

- Increase of the energy consumption of the flying agents and thus reduction of the flight time.
- Position and distance measurement is limited to a certain area around the agents so that approaching agents might be identified too late for collision avoidance.

For these reasons, it is appropriate to connect the UAVs with each other over a communication network that is used to transmit the required information between the agents. As a continuous data exchange increases the energy consumption by transmitters and receivers on the agents and raises the network load, it is reasonable to develop the method in a way that information is only transmitted at certain time instants when new information is necessary. To this aim, communication should only be invoked if appropriate signals violate a threshold.

Theoretical motivation. Achieving the control aims by using only locally measured data and communicated information is a challenging task since the required information may be outdated or missing. The control aims of the networked system have to be split appropriately into local control aims of the individual agents.

For the stabilisation of a system by a control loop that is closed using a communication network the classical event-based control [36, 88] can be used. It aims at reducing the communication effort within a control loop by closing the feedback only at event time instants if the control error violates a threshold. The idea of the event-based control is transferred in this thesis to a higher abstraction level for the trajectory planning of the agents. This approach is challenging due to the following four aspects:

- As communication is invoked aperiodically, the fundamental assumption of discrete-time systems and sampled-data control that the communication time instants or the sampling time instants are equidistant, is violated. Hence, for the event-based communication scheme new models and methods for the description and the analysis of the networked system have to be found.

- As the event-based control method is not used to close a control loop, but it is applied for a trajectory planning, the theory of the event-based feedback control cannot be used. New event-based methods for the trajectory planning and the monitoring of the control aims need to be developed. In contrast to the event-based feedback control where a single event type is sufficient as a clock for the invocation of communication, now five different types of events need to be utilised for communication, trajectory planning and the estimation of the properties of the unreliable communication network.

- For the event-based feedback control events are generated based on current information. In order to plan smooth trajectories for the agents the event-based method to be developed needs to be able to generate events at future time instants. To this aim it uses communicated information that contain the future trajectories of the agents.

- The estimated transmission delay of the network is a statistical quantity. Based on these statistics, deterministic actions have to be executed by the agents to guarantee the control aims. A new method needs to be designed so that deterministic control actions can be generated based on uncertain information.

1.3.2 *Application scenarios*

The event-based control of mobile agents is a concept that can be applied to different types of agents (e.g. UAVs or cars) that have to fulfil tasks individually or in a group by following locally planned trajectories. This section introduces two scenarios in which the method is applied to two UAVs to evaluate the method in simulations and experiments.

Communication relay over aerial base stations. The first scenario considers the communication relay over aerial base stations for the 5G communication [52, 53, 178]. Here, data packets are forwarded over UAVs to ensure requirements on the quality of the communication link. The required channel quality highly depends on the distance $s(t)$ between the agents, which leads to two objectives that need to be fulfilled:

1. Keep a safety distance $\underline{s}$ to ensure a collision-free movement of the agents.

2. Keep a maximum distance $\bar{s}$ between the agents in order to fulfil the requirements on the channel quality.

The thesis focuses on the satisfaction of the requirement $\underline{s} \leq s(t) \leq \bar{s}$ that results from the two objectives. To this aim appropriate trajectories are planned and the agents are controlled along these trajectories. The communication relay is not further considered. The scenario to be considered is illustrated in Fig. 1.3.

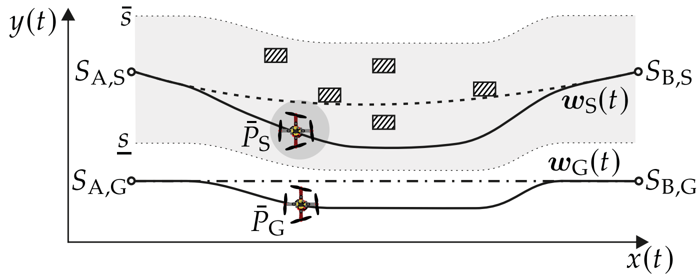

Fig. 1.3: Illustration of the first scenario.

Both agents move on trajectories from their start points $S_{\mathrm{A,S}}$, $S_{\mathrm{A,G}}$ to their end points $S_{\mathrm{B,S}}$, $S_{\mathrm{B,G}}$ on which they should satisfy the requirement $\underline{s} \leq s(t) \leq \bar{s}$ on the current distance $s(t)$ for collision avoidance and the communication relay. One of the agents faces obstacles on its trajectory and changes its trajectory from the dotted line to the solid line to avoid a collision. In this situation, the UAVs act with the following functional associations. The first agent is the stand-on agent $\bar{P}_{\mathrm{S}}$, because it stays on its new trajectory to avoid the obstacles (Fig. 1.3). As a reaction, the second agent is the give-way agent $\bar{P}_{\mathrm{G}}$, because it has to change its locally planned trajectory so as to keep the distance between the UAVs inside the required interval $s(t) \in [\underline{s}, \bar{s}]$.

The problem is formally defined in the next section, where the solution steps are also briefly described.

Formation control of UAVs. In the second scenario a formation of two UAVs should be kept within a tolerance range. A formation is a geometric arrangement of agents that maintain a fixed distance to each other. The formation of UAVs is often utilised for the surveillance of areas [40, 76, 164]. The scenario is illustrated in Fig. 1.4 where the agents move on circular trajectories to enable the experimental evaluation of the scenario within the limited space at the test bed 'MULAN' at the Ruhr University Bochum. Moreover, the generation of circular formations is of relevance for various use cases as in [82, 94, 120, 179]. In this thesis the agents should build a formation that satisfies the following objectives:

1. The agents should be located opposite to one another within a tolerance range defined by γ.
2. The agents should move in different heights z_l and z_h.

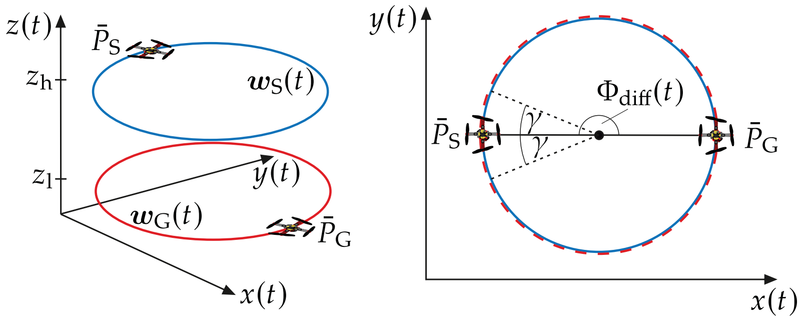

Fig. 1.4: Illustration of the second scenario.

Remark. The agents are located opposite to one another when the angle $\Phi_\mathrm{diff}(t)$ between them satisfies $\Phi_\mathrm{diff}(t) = 180°$. The objectives are similar to the objectives in the first scenario. By considering the distance between the agents, the objectives can also be defined by the distances $\underline{s}$ and $\bar{s}$ of the first scenario. The utilisation of the angle is more convenient to specify the control aims.

As in the first scenario one agent acting as the stand-on agent $\bar{P}_\mathrm{S}$ can change its trajectory $\boldsymbol{w}_\mathrm{S}(t)$ at any time by changing the speed $v_\mathrm{S}(t)$ on the circular path or by changing the height $z_\mathrm{S}(t)$ of its movement. Then, the other agent acting as the give-way agent $\bar{P}_\mathrm{G}$ has to adapt its trajectory $\boldsymbol{w}_\mathrm{G}(t)$ accordingly to keep the objectives satisfied. The problem is formally given in the next section.

Further application scenarios. There are many other use cases for which the method of this thesis can be applied:

- Coordinated search and rescue missions [141, 142, 144].
- Maintenance works on offshore wind farms [55, 77].
- Cooperative movement of heavy loads [96, 162].
- Parcel delivering via drones or autonomous vehicles [91, 155].
- Formations of agents, even formations of satellites [136, 143].
- Merging of vehicle platoons [34, 101].

All application fields have in common that the control aims have to be fulfilled by agents individually or in a group. Furthermore, the control aims can be fulfilled by planning suitable trajectories for the agents, where the agents have to fulfil conditions $\underline{s}$ and $\bar{s}$ on the distance between them.

This means the method enables one a broad range of applications for unmanned mobile agents by planning appropriate trajectories and maintaining application-specific distances $\underline{s}$ and $\bar{s}$.

1.4 Problem formulation and fundamental questions

1.4.1 *Control problem*

The problem is defined for the following two movement scenarios of the agents. First, the agents are allowed to move freely in the 3D space, called *general movement* of the agents. Second, the agents are limited to circular trajectories in two different heights, named *circular movement* of the agents.

Problem statement for agents with a general movement. A networked control system that consists of two agents, which move on trajectories in the 3D space and should satisfy individual tasks as well as cooperative tasks has to fulfil the following requirements:

(A1) **Trajectory planning and monitoring:** The trajectories $\boldsymbol{w}_l(t)$ of the agents $\bar{P}_l$, $(l \in \{S, G\})$ should bring them from their start points $S_{A,l}$ to their end points $S_{B,l}$. The separation $s(t) = ||\boldsymbol{p}_G(t) - \boldsymbol{p}_S(t)||$, given by the euclidean vector norm defined in (2.1) on p. 29 should satisfy

$$\underline{s} \leq s(t), \quad \forall t \tag{1.1}$$

$$s(t) \leq \bar{s} \tag{1.2}$$

where $\underline{s}$ is a safety distance to avoid collisions between the agents and $\bar{s}$ is a maximum separation to achieve specific control aims (e.g. communication relay).

(A2) **Trajectory following:** The agents $\bar{P}_l$, $(l \in \{S, G\})$ should follow given trajectories $\boldsymbol{w}_l(t)$ exactly if there are no external disturbances:

$$\boldsymbol{p}_l(t) = \boldsymbol{w}_l(t), \quad l \in \{S, G\}, \quad \forall t.$$

(A3) **Disturbance compensation:** External disturbances (e.g. wind) should be compensated by the agents to ensure a smooth flight behaviour.

The control method is supposed to respect the following constraints:

(C1) **Network topology:** There is no coordinator or forwarding unit in the network but all control and estimation algorithms have to work with local information.

(C2) **Communication:** The communication link between the agents should only be invoked if the uncertainty of the local information exceeds a threshold.

(C3) **Computation:** The computation power onboard the agents is limited and causes bounded computational delays.

Problem statement for agents with a circular movement. If the agents move on circular trajectories, the control aims and the constraints are the same as above, but due to the restriction of the direction of the movement, specific methods can be developed. As the circular trajectories are represented in the cylindrical coordinate system, it is more convenient to specify the control aims based on the angle difference $\Phi_{\mathrm{diff}}(t)$ between the agents as well as on the height difference $z_{\mathrm{diff}}(t)$. Therefore, for a circular movement of the agents, requirements (1.1) and (1.2) are replaced by:

$$\Phi_{\mathrm{diff}}(t) \in \left[\underline{s}, \bar{s}\right], \quad \forall t \tag{1.3}$$

with

$$\underline{s} = 180^\circ - \gamma \tag{1.4}$$

$$\bar{s} = 180^\circ + \gamma \tag{1.5}$$

and

$$z_{\mathrm{S}}(t), z_{\mathrm{G}}(t) \in [z_{\mathrm{l}}, z_{\mathrm{h}}], \tag{1.6}$$

which causes the height difference

$$z_{\mathrm{diff}}(t) = |z_{\mathrm{S}}(t) - z_{\mathrm{G}}(t)| = \bar{z} \quad \text{with} \quad \bar{z} = |z_{\mathrm{h}} - z_{\mathrm{l}}|. \tag{1.7}$$

Relation (1.3) claims the agents to be located opposite to one another within a tolerance range specified by γ. Equation (1.7) demands that the agents keep a height difference to one another specified by the two allowed heights z_{l} and z_{h} given in (1.6). This requirement should hold whenever the agents move on a constant height, but not during height changes of the agents.

The thesis focusses on the main challenge to develop a method for the planning of trajectories for the agents so as to satisfy control aim (A1). In Chapter 10 a flatness-based two-degrees-of-freedom controller is derived with which control aims (A2) and (A3) are satisfied.

1.4.2 Decomposition of the control problem and steps of solution

Allocation of the control aims of the networked control system into local aims of the agents. In order to develop a method that is executed locally by the agents and that leads to the fulfilment of the control aims (A1) – (A3) by respecting the constraints (C1) – (C3), the control aims of the networked control system have to be subdivided into local control aims of an individual agent. Satisfaction of the local control aims leads to the fulfilment of the control aims of the overall system.

In order to satisfy control aim (A1) the agents have to fulfil their individual control aims, which are named (S1) for the stand-on agent and (G1) for the give-way agent. As the two agents are assigned with different functions, their local aims differ:

(S1) **Trajectory planning and monitoring:** The trajectory $\boldsymbol{w}_{\mathrm{S}}(t)$ of the stand-on agent should bring it from its start point $S_{\mathrm{A,S}}$ to its end point $S_{\mathrm{B,S}}$. It should equal a desired trajectory $\boldsymbol{w}_{\mathrm{S,d}}(t)$ to fulfil its individual task and to avoid collisions with obstacles:

$$\boldsymbol{w}_{\mathrm{S}}(t) = \boldsymbol{w}_{\mathrm{S,d}}(t), \quad \forall t.$$

(G1) **Trajectory planning and monitoring:** The trajectory $\boldsymbol{w}_{\mathrm{G}}(t)$ of the give-way agent should bring it from its start point $S_{\mathrm{A,G}}$ to its end point $S_{\mathrm{B,G}}$. It should be planned to fulfil the following requirements:

- The trajectory $\boldsymbol{w}_{\mathrm{G}}(t)$ should coincide with a desired trajectory $\boldsymbol{w}_{\mathrm{G,d}}(t)$ to fulfil its individual task and to avoid collisions with obstacles

$$\boldsymbol{w}_{\mathrm{G}}(t) = \boldsymbol{w}_{\mathrm{G,d}}(t), \quad \forall t.$$

- The separation $s(t)$ fulfilling (1.1) and (1.2) should be guaranteed.

If the trajectory is changed at a time instant t_{c} to fulfil (1.1) or (1.2), the agent should return to the trajectory $\boldsymbol{w}_{\mathrm{G,d}}(t)$ as soon as it is possible at a time instant

$$t_{\mathrm{c}+1} = \arg\min_{t > t_{\mathrm{c}}} \left\{ \underline{s} \leq s(t) \wedge s(t) \leq \bar{s} \right\}.$$

The control aims (A2) and (A3) correspond unchanged to the local aims of the agents. The control aims are renamed to (S2), (G2) and (S3), (G3) for consistency. Likewise, the individual agents are subject to the constraints (C1) – (C3).

Remark. The control aim (G1) can be partly contradictory in the sense that the give-way agent might not be able to follow the desired trajectory $\boldsymbol{w}_{\mathrm{G,d}}(t)$ and to comply with the conditions (1.1) and (1.2) at the same time. As the conditions (1.1) and (1.2) are of superior priority, the give-way agent has to neglect following the trajectory $\boldsymbol{w}_{\mathrm{G,d}}(t)$.

Analysis of the local control aims and fundamental questions. An analysis of the local control aims leads to the fact that the stand-on agent can change its trajectory at any time. As communication is invoked only at event time instants using an unreliable network that induces transmission delays and packet losses the following questions arise that need to be addressed by the event-based method:

- How can the time delay of the information transmission be estimated?
- At which time instant communication needs to be invoked with respect to time delays and packet losses caused by the unreliable communication network?
- Which information does the individual agent need to plan appropriate trajectories locally?
- How can deterministic control actions be generated based on uncertain information?
- How does the trajectory need to be planned to satisfy different control aims?

The solution to these issues enables the agents to fulfil the control aims.

Main problems. Considering the analysis of the local control aims the problems are identified, which need to be solved by the agents to fulfil their control aims. In the thesis it is focussed on the problems that need to be solved by the agent acting as the give-way agent, because it is responsible to fulfil the requirements (1.1) and (1.2) of control aim (A1) or the requirements (1.3) and (1.6) of control aim (A1) for a circular movement. The stand-on agent just has to follow its planned trajectory. Four problems are determined, which have to be solved by the give-way agent:

- An estimation method needs to be developed to derive an estimate of the current QoS properties of the communication channel, because the channel quality varies with the relative movement of the agents to one another.
- A method has to be found to estimate the future positions of the stand-on agent, because the communicated trajectory becomes uncertain over time.
- A method needs to be derived to determine the uncertainty of the local data in order to decide when it is necessary to invoke communication and when to change the trajectory. To this aim appropriate event thresholds have to be found with respect to the uncertain QoS properties of the network and the dynamics of the agents.
- A trajectory planning method has to be developed to get trajectories that respect the dynamics of the agents and fulfil the control aims.

The problems are given in detail below. The method uses two notations $(t_k, \tilde{t}_i)$ to state two different time instants at which events are generated by the event-based control units and a new estimate of the properties of the communication channel is derived by the delay estimators. The notation is stated in detail in Chapter 8. Furthermore, the distance between the agents is denoted by $s(t)$ for the control aims and by $d(t)$ in the context of estimating the properties of the network.

Problem 1.1. (Time delay estimation)

Given:	• Parametrised Markov model describing the channel properties. • Maximum distance $\max \mathrm{dist}(\boldsymbol{w}_{\mathrm{G}}(t), \mathcal{P}_{\mathrm{S}}(t, t_{\mathrm{r},k}, \tau_k))$ between the trajectory of the give-way agent and the possible positions of the stand-on agent defined in Problem 1.2 in a time interval $t \in [\tilde{t}_i, \tilde{t}_{i+1}]$.
Find:	Statistical estimate $\tilde{\tau}_{\max}(d(\tilde{t}_i))$ of the mean time delay $\tau_k = \tau_{\mathrm{c}} + \tau_{\mathrm{n}}$ in a time interval $[\tilde{t}_i, \tilde{t}_{i+1}]$ consisting of the computation time τ_{c} of the agents and the transmission delay τ_{n} of the network.
Boundary conditions:	Channel statistics are constant if the agents remain in a distance of at most $\bar{d}$ to one another (Assumption 8.1 on p. 175).

Problem 1.2. (Position estimation)

Given:	• Communicated position $\boldsymbol{p}_{\mathrm{S}}(t_{\mathrm{c},k})$ and speed $\boldsymbol{v}_{\mathrm{S}}(t_{\mathrm{c},k})$ of the stand-on agent at the communication time instant $t_{\mathrm{c},k}$. • Communicated trajectory $\boldsymbol{w}_{\mathrm{S}}(t)$, $(t_{\mathrm{c},k} \leq t \leq t_{\mathrm{end}})$ of the stand-on agent. • Estimated time delay $\tilde{\tau}_{\max}(d(\tilde{t}_i))$. • Mean time delay τ_k. • Time instant $t_{\mathrm{r},k}$ of reception of information.
Find:	A set $\mathcal{P}_{\mathrm{S}}(t, t_{\mathrm{r},k}, \tau_k)$ that includes the future positions of the stand-on agent such that $$\boldsymbol{p}_{\mathrm{S}}(t) \in \mathcal{P}_{\mathrm{S}}(t, t_{\mathrm{r},k}, \tau_k), \quad t \geq t_k.$$
Boundary conditions:	• System and actuator limitations given by (1.8), (1.9) on p. 17. • Maximum speed (1.10) or (1.11) on p. 17.

Problem 1.3. (Uncertainty estimation of the local data and invocation of communication)

Given:
- Trajectory $\boldsymbol{w}_{\mathrm{G}}(t)$, $(t_{\mathrm{c},k} \leq t \leq t_{\mathrm{end}})$ of the give-way agent.
- Inclusion $\mathcal{P}_{\mathrm{S}}(t, t_{\mathrm{r},k}, \tau_k)$ of positions of the stand-on agent.
- Estimated time delay $\tilde{\tau}_{\max}(d(\tilde{t}_i))$.

Find: Event threshold $\bar{e}_{\mathrm{G}}$ to determine communication time instants $t_{\mathrm{c},k}$ and event time instants t_k. Communication must be invoked, so that the required data can be received at t_k, $(t_k > t_{\mathrm{c},k})$ under consideration of $\tilde{\tau}_{\max}(d(\tilde{t}_i))$.

Boundary conditions: Adherence of a minimum time span between two consecutive events.

Problem 1.4. (Trajectory planning)

Given:
- Communicated trajectory $\boldsymbol{w}_{\mathrm{S}}(t)$, $(t_{\mathrm{c},k} \leq t \leq t_{\mathrm{end}})$ of the stand-on agent.
- Trajectory $\boldsymbol{w}_{\mathrm{G}}(t)$, $(t_{\mathrm{c},k} \leq t \leq t_{\mathrm{end}})$ of the give-way agent.

Find: Trajectory for the give-way agent so that control aim (G1) remains fulfilled.

Boundary conditions:
- System and actuator limitations given by (1.8), (1.9) on p. 17.
- Maximum speed (1.10) or (1.11) on p. 17.

Remark. In the case the agents are connected by an ideal communication network that does not induce any transmission delays or packet losses, communicated information is always received by the agents instantaneously $(t_{\mathrm{c},k} = t_k)$ and the give-way agent does not need to solve Problem 1.1.

Structure of the event-based control unit A**.** The four problems are solved by the agents using a delay estimator D and an event-based control unit A. In Fig. 1.5 the basic structure of the units A_{G} and D_{G} of the give-way agent is shown as the give-way agent is responsible to satisfy control aim (A1). The structure of the control units A_{S} and D_{S} of the stand-on agent is identical, but they execute different tasks. Solid arrows depict continuous signal transmissions while the dashed arrows represent event-based signal transfers.

The delay estimator D_{G} and the control unit A_{G}, which consists of three parts, execute the following tasks to solve the Problems 1.1 – 1.4. The estimator D_{G} generates the statistical estimate $\tilde{\tau}_{\max}(d(\tilde{t}_i))$ of the time delay using a two-state Markov model based

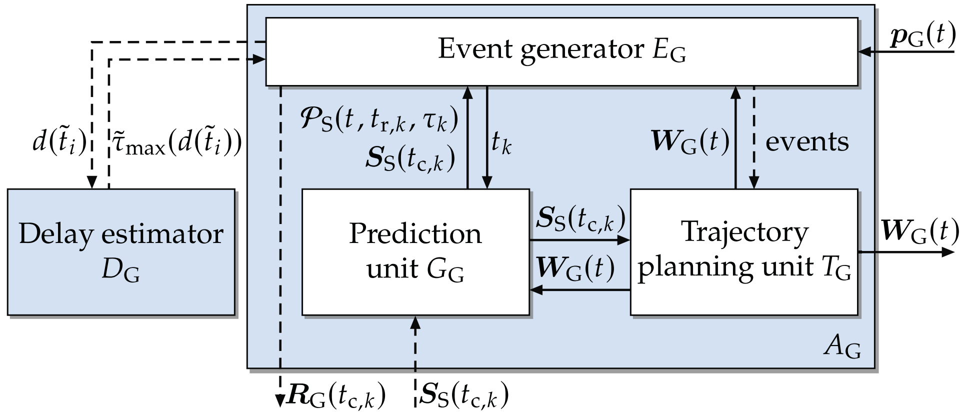

Fig. 1.5: Structure of the event-based control unit A_G with the delay estimator D_G of the agent $\bar{P}_G$.

on the maximum distance $d(\tilde{t}_i)$ between the agents (Problem 1.1). The *prediction unit* G_G receives the communicated data $\boldsymbol{S}_S(t_{c,k})$ introduced on p. 129 of the stand-on agent and generates an inclusion $\mathcal{P}_S(t, t_{r,k}, \tau_k)$ of the position of the stand-on agent between two consecutive event time instants t_k, t_{k+1} (Problem 1.2). The *event generator* E_G supervises the fulfilment of the control aims. It uses the estimated time delay $\tilde{\tau}_{max}(d(\tilde{t}_i))$, the current position $\boldsymbol{p}_G(t)$ of the give-way agent and the matrix $\boldsymbol{W}_G(t)$, which contains its trajectory to invoke communication by generating an event and by sending a request $\boldsymbol{R}_G(t_{c,k})$ introduced on p. 195 (Problem 1.3). It triggers the time delay estimation whenever the current estimate becomes too uncertain. Furthermore, it decides when it is necessary to change the trajectory of the give-way agent by generating an event. The *trajectory planning unit* T_G changes the trajectory appropriately in order to keep control aim (G1) fulfilled (Problem 1.4).

1.4.3 *Main assumptions*

This paragraph states the assumptions that apply throughout the thesis. It should be noted that Assumptions 1.2 and 1.3 concern the possible movements of the UAVs, where Assumption 1.2 relates to the general movement of the agents and Assumption 1.3 applies to the circular movement. In this thesis the method is applied to the control of quadrotors. Hence, these assumptions are made with respect to the dynamics of the quadrotors.

Assumptions regarding the properties of the communication network are made in Chapters 6 and 8.

Assumption 1.1. The agents are only able to measure their local data as position, speed and acceleration.

Assumption 1.2. The dynamics of the quadrotors are restricted by

- state limitations represented by the angles $\phi(t)$ and $\vartheta(t)$ around the axes of the quadrotor

$$\begin{aligned} \underline{\phi} \leq \phi(t) \leq \bar{\phi} \\ \underline{\vartheta} \leq \vartheta(t) \leq \bar{\vartheta}. \end{aligned} \tag{1.8}$$

- actuator limitations of the rotor speeds n_i

$$\underline{n} \leq n_i \leq \bar{n}, \quad i = 1, \dots, 4. \tag{1.9}$$

The speeds $\boldsymbol{v}_{\mathrm{G}}(t)$ and $\boldsymbol{v}_{\mathrm{S}}(t)$ of the give-way agent and the stand-on agent are bounded:

$$||\boldsymbol{v}_{\mathrm{G}}(t)|| = \left\| \frac{\mathrm{d}\,\boldsymbol{w}_{\mathrm{G}}(t)}{\mathrm{d}\,t} \right\| \leq v_{\mathrm{G,max}} \quad \text{and} \quad ||\boldsymbol{v}_{\mathrm{S}}(t)|| \leq v_{\mathrm{S,max}} = v_{\mathrm{max}}. \tag{1.10}$$

In case the give-way agent has to plan a reactive trajectory introduced in Section 7.4, it is able to increase its speed so that

$$v_{\mathrm{S,max}} < v_{\mathrm{G,max}} = 1.5\, v_{\mathrm{max}}$$

holds.

Assumption 1.3. The quadrotors move on the circular path constantly in one direction of rotation (mathematically positive or negative direction) and the circular paths are only planned in the two different heights z_{l} and z_{h}. The speeds $\boldsymbol{v}_{\mathrm{S}}(t)$ and $\boldsymbol{v}_{\mathrm{G}}(t)$ of $\bar{P}_{\mathrm{S}}$ and $\bar{P}_{\mathrm{G}}$ are bounded:

$$\begin{aligned} v_{\mathrm{S,min}} \leq ||\boldsymbol{v}_{\mathrm{S}}(t)|| \leq v_{\mathrm{S,max}} \\ v_{\mathrm{G,min}} \leq ||\boldsymbol{v}_{\mathrm{G}}(t)|| \leq v_{\mathrm{G,max}} \end{aligned} \tag{1.11}$$

with $v_{\mathrm{G,max}} > v_{\mathrm{S,max}}$ and $v_{\mathrm{G,min}} < v_{\mathrm{S,min}}$. The difference of the maximum speeds and the difference of the minimum speeds is identical:

$$|v_{\mathrm{S,min}} - v_{\mathrm{G,min}}| = |v_{\mathrm{S,max}} - v_{\mathrm{G,max}}|.$$

Assumption 1.4. The trajectories of the quadrotors are planned using piecewise Bézier curves introduced in the next chapter. According to Assumption 1.2 the conditions on the start points and the end points of the trajectories should satisfy:

$$\begin{aligned} &\boldsymbol{w}(\bar{t}_i) = \boldsymbol{0}, \quad ||\dot{\boldsymbol{w}}(\bar{t}_i)|| \leq v_{\mathrm{max}}, \quad ||\dot{\boldsymbol{w}}(\bar{t}_{i+1})|| \leq v_{\mathrm{max}}, \\ &\ddot{\boldsymbol{w}}(\bar{t}_i) = \ddot{\boldsymbol{w}}(\bar{t}_{i+1}) = \boldsymbol{w}^{(3)}(\bar{t}_i) = \boldsymbol{w}^{(3)}(\bar{t}_{i+1}) = \boldsymbol{w}^{(4)}(\bar{t}_i) = \boldsymbol{w}^{(4)}(\bar{t}_{i+1}) = \boldsymbol{0}, \end{aligned} \tag{1.12}$$

which can be easily achieved by a coordinate shift.

1.5 Literature review

The proposed method combines approaches from the fields of communication technology and control theory. Communication technology provides models of wireless channels for different application fields that depend on the environment the agents are moving in (e.g. urban or rural environment). With these models it is possible to describe the quality-of-service (QoS) parameters of the channels. Control theory contributes methods concerning the position estimation of moving agents, the event-based control of feedback systems, the event-based communication and the trajectory planning for moving agents for various application cases.

This literature review gives an overview over the relevant methods from both fields that are utilised to develop the event-based control method for mobile agents in this thesis.

Modelling of wireless communication channels. Networked control systems have become increasingly popular, because they can be applied in many areas (e.g. connected vehicles, networked UAV operations). The classical control theory has been focussed on the control of these systems under the assumption that the communication network has ideal properties in the sense that it does not induce transmission delays or packet losses. However, this assumption is not valid for systems that use a wireless network with inherent imperfect channel properties or a limited bandwidth as in this thesis. Hence, the channel properties need to be considered for the control with an appropriate model. There are many approaches to model wireless channels for different applications in the literature [103, 163].

Channel models for UAV applications as in this thesis can be subdivided by the propagation scenario into air-to-air channels and air-to-ground channels. The air-to-air propagation is an important aspect for the communication between UAVs. The characteristics of these channels depend mainly on the environmental conditions, UAV flight direction, the existence of a line-of-sight (LOS) path between the UAVs and ground reflections [32, 81, 97, 170]. The models focus either on the large-scale statistics such as path loss and shadowing [49, 151, 174] or on the small-scale statistics that are modelled with various models such as Rayleigh, Rice or Nakagami [31, 102, 150, 182]. These fading amplitude statistics are important to determine the random behaviour of fading channels using the first order statistics given by a cumulative distribution function (CDF) and a probability density function (PDF). The air-to-ground propagation models the communication between an UAV and ground stations and is not further considered in this thesis.

Furthermore, analytical channel models are utilised to model the propagation behaviour of a channel under certain assumptions and parameters. The models are derived with three approaches: deterministic, stochastic and geometry-based [67, 123, 131].

An approach to derive the QoS parameters of a channel, which can be utilised for the control algorithms, is the use of the effective capacity theory. Here, a link layer model is stated in which it is possible to derive the required data rate of a channel to satisfy given QoS parameters of a physical channel. This method was extended to different source rates and a packetised communication in [168, 169].

In this thesis the Rice fading model is utilised to represent the channel statistics. It is used by the delay estimator to generate an estimate of the current transmission delay of the channel.

Position estimation approaches. The position estimation of moving agents especially of UAVs is a field of growing interest. The motivation is the fact that complex tasks can be solved by the cooperation of several agents. However, this requires that the individual agent is aware of the relative positions and orientations of neighbouring agents to determine the distances to these agents. To this aim, the agent should be localised in the same coordinate frame. In this thesis the future positions of the stand-on agent are estimated, because its future trajectory is uncertain. The positions of the stand-on agent need to be known to ensure the collision avoidance.

Many approaches for a position estimation use image-based algorithms, which process the data from onboard cameras or incorporate measurements of GPS sensors [35, 59, 104–106, 128, 171]. In addition, commercial drones utilise real-time kinematics [90, 159]. This is a method of precisely determining position coordinates using satellite navigation techniques. Different approaches for the position estimation have been proposed in [48, 71, 118], where a Kalman filter is applied to estimate the position of agents by analysing the received signal strength from the onboard communication module, the angle of arrival or the homography of a series of images. In [160] the possible positions of ground moving agents have been included in a circular set that expands over time.

A different field of application is to estimate the future positions of agents in a disturbed environment. For collision avoidance it is necessary that an agent is aware of the set of possible future positions of nearby agents in order to be able to plan guaranteed collision-free trajectories. An approach is to analyse the past movement to get a reliable estimate of the future position and path of an agent [38]. Different approaches make use of a known trajectory of an agent and estimate the future positions to be in a tube around this trajectory when the agent is affected by external disturbances [175, 181].

In this thesis the idea from [160] is used. The approach is extended to estimate the position of an agent in the 3D space and the conservatism of the estimate is reduced compared to [160]. Furthermore, with the method in this thesis the future positions of an agent can be estimated.

Event-based control and communication. Networked control systems are control systems, where the components (sensors, actuators and controllers) are connected by a communication network. As the network might be used by several components of a system at the same time it is reasonable to reduce the amount of communicated data as far as this is possible without threatening the control aims. The control approach in this thesis utilises a communication scheme where information between the agents is only exchanged if it is necessary.

At an early stage periodic control schemes (also called time triggered control schemes) are developed due to the ease of design and analysis [62]. Here, information is communicated with a fixed sampling period, which is often chosen to be small in order to guarantee the desired system performance. In order to mitigate the unnecessary waste of network capabilities the event-based control scheme (or event-triggered control scheme) was introduced in [36, 37, 88]. By using this approach control tasks are executed after an event is generated when an appropriately chosen threshold is violated instead of elapsing a time span [87]. The reduction of resource utilisation motivates the wide range of applications of the event-based communication in state-feedback control [86, 133, 158], output-feedback control [63, 64] and formation control and consensus of multi-agent systems [70, 148, 166].

The event-based control approaches can be classified into three categories: event-triggered sampling schemes [44, 63, 64, 74, 117, 156], self-triggered sampling schemes [61, 87, 124] and discrete event-triggered communication schemes [58, 69, 78]. In the event-triggered sampling the state of a system is constantly supervised. An event is generated and a new control input is determined whenever a predefined threshold is violated. The threshold is state-dependent and can be given by a fixed value or by a function. In multi-agent systems where global information may be unavailable for each local system decentralised event-triggered control schemes are proposed that work with locally measured data and communicated information [63, 156]. Using the self-triggered sampling scheme the next event time instant is determined by only evaluating previously received data and utilising knowledge about the dynamics of the system. The state of the system to be controlled is not constantly measured. Due to the over-approximation of the state of the system, often more events are generated compared to the event-triggered control scheme. Both control schemes should guarantee a positive minimum inter-event time span (MIET). Otherwise Zeno behaviour occurs that is generating an infinite number of events in a finite period of time, which renders the control approaches to be infeasible [44]. For several event-triggered control schemes no positive MIET can be guaranteed in the presence of arbitrarily small disturbances [44]. In these cases a discrete event-triggered communication scheme can be applied. Here, the state of the system is sampled periodically while the event condition is evaluated only at a sampling time instant, which excludes Zeno behaviour.

As there are many communication imperfections in networked control systems, such as transmission delays, packet losses, communication constraints or quantisation effects, the event-triggered control scheme is extended with many approaches to cope with these uncertainties. Often a bound on the transmission delay has been stated so that the event-based state-feedback control loop is stable despite using the delayed information or machine learning techniques have been applied to compensate the random delays [79, 111, 176]. In [80, 115] communication protocols have been proposed that use acknowledgement (ACK) messages to deal with delays and packet losses. The idea is to send an ACK message after the reception of a data packet. If the sending agent does not receive an ACK message from all other agents, it resends its data to those agents, which did not send the ACK message.

Most of the approaches in the literature have focussed only on some aspects of the communication imperfections while ignoring the others. In [89] a hybrid system framework has been stated that incorporates communication constraints, varying transmission intervals and varying delays to guarantee stability of a networked control system based on Lyapunov functions.

This thesis utilises the idea of the event-triggered sampling to invoke communication and a change of the trajectory of the agent. In addition the method to be developed is able to generate events at future time instants by an evaluation of the communicated trajectories.

Motion planning and path following. Motion planning is an indispensable part for autonomously moving agents. Motion planning is responsible for coordinating the overall system by interacting with human operators and making decisions in environments with obstacles. Hence, the motion planning is often referred to as the highest layer of an autonomous system. As the method is applied to quadrotors in this thesis, this literature review overviews motion planning approaches for UAVs. However, the basic principles can also be applied to other autonomous agents. The planning problem can be roughly divided into two steps. First, a discrete path must be found and second, a trajectory is generated, which is often planned using optimisation methods.

In order to find suitable paths, methods have been proposed ranging from sampling-based [107] to searching-based [113]. For applying these methods a configuration space has to be defined [108]. It is a set of possible transformations that can be performed by the agents. Using the configuration space, motion planning problems that vary in geometry and kinematics can be solved by the same planning algorithms.

The two most common sampling-based methods are the probabilistic road-map (PRM) [149] and the rapidly-exploring random tree (RRT) [109]. The PRM method picks random samples from the free space of the agent and uses a local planning unit to connect the new samples to its nearby configurations. It results a graph model, where the motion planning problem is solved using graph search algorithms. The RRT method consists of a tree which root is the start configuration. Each tree node is a configuration in the free space. The RRT grows by iteratively adding edges by random configurations if the connection between the random configuration and the nearest tree node is feasible. The random configuration becomes a node and an edge is added between it and the nearest tree node. Once the tree has extended to a configuration near the target configuration, it is terminated and a path is immediately generated. The RRT method efficiently finds a suitable path. However, it does not find the optimal path but converges to the first suitable solution. By extending the method, it is also possible to find an optimal solution [100].

Searching-based methods discretise the configuration space and convert the path finding problem to a graph searching problem. Hence, the widely used algorithms as the breadth-first search or the depth-first search can be applied [57]. While the breadth-first search provides an optimal solution if the graph is a uniform-weighted graph, with the depth-first search an optimal solution cannot be guaranteed [173]. Furthermore, the Dijkstra algorithm can be applied to derive the shortest path and the A^* algorithm is used to find a least-cost path [57].

As most path finding algorithms generate a path without time information, the path needs to be parametrised in time to obtain a trajectory. This is the main objective of the trajectory generation. The trajectory generation problem is formulated as minimising an objective function such as the control cost while fulfilling constraints on the dynamics of the agent. The methods can be categorised as hard-constraint [127, 138] and soft-constraint [146, 183]. The hard-constraint methods treat all constraints on the trajectory in the same way. In contrast, the soft-constraint methods penalise the constraints in the objective function so as to obtain better results especially in noisy environments.

In order to enable a quadrotor to follow the planned paths, several methods are developed. One of the most common control techniques for UAVs is the feedback linearisation. The aim of this control approach is to linearise the system in a certain operation point by applying a non-linear inversion of the system. The non-linearities are erased and the linear control theory can be applied [33, 47]. Another approach is based on the Lyapunov theory. Adherence of the Lyapunov stability condition leads to the convergence of the controller [56]. Furthermore, model predictive control (MPC) can be used for the path following. Here, the control problem is transformed into an optimisation problem [50, 68] and solved iteratively for a time horizon. In [130] a vector field has been utilised to make a quadrotor follow its path.

In this thesis the motion planning is used to determine collision-free trajectories with respect to static obstacles. In Chapter 10 a controller is provided that ensures the path following, which is necessary for guaranteeing the collision avoidance.

Trajectory planning for different applications. Trajectories for moving agents are planned to meet specific control aims. In this thesis trajectories are planned in order to achieve a collision-free movement and to provide wireless coverage for moving ground objects. This literature review gives a brief overview of approaches to satisfy the aforementioned two control aims.

The problem of collision avoidance has been addressed with several approaches that can be roughly subdivided into two categories: reactive methods and proactive methods. The reactive methods react on a detected collision and change the movement of an agent to avoid the collision. Some of the methods do only change the speed or the acceleration of the agent along its path that remains unchanged. Such methods use artificial potential fields [112, 121] or velocity obstacles [54, 126]. Another approach is to use a sampling-based check of the feasibility of the current trajectory and to change it when a collision is detected [114]. In terms of sensory requirements vision-based approaches achieve good results, where the line-of-sight angle and the time-to-collision between two agents is taken into account [122]. In contrast, the proactive methods detect a collision in advance and plan the trajectories so that they are collision-free. This requires the complete knowledge of all trajectories of nearby agents and obstacles or an appropriate bound on the uncertainty of the trajectory. Collision-free trajectories can be planned based on Bézier curves [125, 154] or by utilising the receding horizon based method of the MPC [39]. In [30] collision-free paths have been planned with respect to communication constraints.

The approach presented in thesis combines both the proactive and the reactive method. If a collision is detected in the future, the trajectory of the give-way agent is changed in advance for a collision avoidance. In contrast to the methods described above, the method is able to change the trajectory at runtime. If a collision is imminent the method reacts instantaneously with a rapid change of the trajectory to avoid the collision.

In recent years, there has been an increasing interest in using UAVs to provide wireless coverage for 5G communication or in emergency cases where the UAVs act as aerial relay stations [46]. An approach is to optimise the trajectory of an UAV that is capable to move with high speeds to maximise the end-to-end throughput [140, 177]. In [52, 93] an optimal position for an aerial base station has been derived to maximise the coverage range of a single UAV.

The method developed in this thesis does not aim at achieving the largest coverage but at providing moving ground objects communication with a specified channel quality.

Formation control of agents. In a second scenario the event-based control method is applied to keep a formation of agents. The problem of formation control of UAVs has been widely investigated. Different approaches in the literature include the leader-follower control [83, 85], the behaviour-based control [110] and the virtual structure method [180]. For the leader-follower control, the formation is obtained with a controller that depends on a single leader state. The formation with the behaviour-based approach is obtained by weighting the control inputs of several agents. For the virtual structure method the formation of agents is considered as a single agent in the virtual structure. This limits the field of application as only the movement of one agent can be controlled. The mentioned approaches can be categorised into position-based, displacement-based and distance-based control considering the sensing capabilities and the amount of information exchange between the agents. In [99] an event-based communication scheme has been proposed for the formation control of UAVs. In [53] a circular trajectory has been planned for an UAV acting as an aerial communication relay station. The method aims at reducing the energy for the flight manoeuvres and the communication relay.

In this thesis two quadrotors build a formation while moving on circular trajectories.

1.6 Contribution of this thesis

The approach presented in this thesis aims at avoiding collisions and keeping a maximum separation between two agents. It uses only local data and communicated information over an unreliable network, where communication is only invoked if it is necessary for the satisfaction of the control aims. The control method combines approaches from both, control theory and communication technology, to improve the control results. The main contributions of the thesis with respect to developing an event-based control method for the agents are as follows.

Trajectory planning. The agents follow locally planned trajectories to fulfil the control aims. As the agents should be able to fulfil different tasks, different trajectories need to be planned to fulfil them. In the literature often trajectory planning methods are stated that are able to fulfil one specified control task (e.g. guarantee collision avoidance) but they are not able to fulfil various tasks. Hence, this thesis develops a trajectory planning method that consists of two parts: First, in the planning task part the boundary conditions are specified for a trajectory to fulfil the control aims. Second, the planning algorithm part determines the trajectory with respect to the planning task and the dynamics of the agent. This approach enables the agents to fulfil different control tasks by only changing the trajectory planning task, while the planning algorithm remains unchanged.

The main result is a trajectory planning method that generates trajectories for the agents so that they are able to fulfil different control tasks.

Event-based communication. The communication between the agents is event-based in the sense that information is only transmitted when the uncertainty about the locally available data becomes too large. For the communication scheme the idea is utilised that data only needs to be sent when it is necessary as performed in the event-based feedback control. In the literature the event-based control is utilised to stabilise systems where the feedback loop is closed over a communication network. It aims at reducing the network load. For the control of the mobile agents a method is developed that realises the idea of the event-based control on a higher abstraction level to use it for a trajectory planning. Communication is invoked whenever a threshold is violated that represents the uncertainty about the local data. Furthermore, the method utilises thresholds to decide when the movements of the agents need to be changed so as to fulfil the control aims. The thresholds are determined with respect to the dynamics of the agents and consider the estimate of the current time delay of an information transfer.

The main result is an event-based method that invokes communication only when the uncertainty about the local data becomes too large and that invokes a change of the trajectories of the agents only when it is necessary. It takes the estimated time delay into account to generate appropriate control actions and generates events at future time instants in order to ensure the control aims.

Estimation of positions of an agent. For the determination when communication has to be invoked or the trajectory of the give-way agent needs to be changed, the distance between the agents has to be known. This means the position of the neighbouring agent must be known continuously.

In contrast to most approaches in the literature no sensors for a distance measurement are used in this thesis and communication takes place only at discrete time instants. Additionally the trajectory of the neighbouring agent is uncertain, which requires a prediction of the possible positions of the neighbouring agent between two event time instants. Hence, the thesis presents a method that generates an ellipsoidal inclusion of all possible future positions of a nearby agent between two event time instants. The inclusion is only based on the position and the speed of the agent at the last event time instant and is less conservative compared to the approach in [160] since it incorporates the dynamics of the agents.

The main result is an estimation method that provides an inclusion of future positions of an agent with respect to its dynamics.

Estimation of network properties. Communication is invoked with an event-based scheme only at discrete time instants when an agent needs new information. This requires that information is received immediately by the agent. However, using an unreliable communication network causes transmission delays and packet losses, so that information may be received late by an agent or it is even lost. Furthermore, the limited computational power onboard the agents results in computational delays when processing the data. Hence, these time delays must be taken into account when determining the event time instants.

In the literature of communication technology the approaches focus often only on the modelling of a wireless channel, while in the literature of control theory the results are often achieved by using fixed upper bounds on the time delay, which makes the results conservative. Therefore, this thesis uses a channel model from the literature of communication technology that represents the properties of the network. An event-based method is introduced that determines the transmission delay induced by the network using the channel model. Furthermore, it generates an estimate of the current time delay consisting of the transmission delay and the computational delay caused by the limited computational power of the agents. This variable estimate that depends on the movement of the agents is used for determining the next communication time instants. In order to obtain an accurate estimate of the time delay over time the method updates the channel model depending on the movement of the agents.

The main result is a method that estimates the current time delay of an information transfer. It is used for the invocation of communication in the presence of these delays.

1.7 Structure of this thesis

The thesis consists of five parts. Part I introduces the control problem and the mathematical basics for the event-based control method. Part II gives the basic methods needed for the event-based control of the agents. In Part III, these methods are applied for the control of agents that are connected over an ideal communication network. This part focusses on the description of the parts of the control units. In Part IV the methods are extended to ensure the control aims for agents communicating over an unreliable network. The focus of this part is to show the impact of the methods from communication technology on the control methods. Part V states the simulation results and the experimental results for an application of the method to two quadrotors.

Chapter 2 introduces the notation, which is used throughout the thesis. The mathematical basics of Bézier curves that are used for the trajectory planning are stated. Relevant algorithms are given that are utilised to determine the distance between two Bézier curves. Two demonstration examples are described, which are used as running examples in order to illustrate the parts of the control method.

Chapter 3 presents the trajectory planning method. Different methods are given to plan trajectories in the cartesian coordinate system and to plan circular trajectories in the cylindrical coordinate system. Furthermore, the planning of trajectories to change the speed on the circular path or the height of the circular movement are stated.

Chapter 4 proposes the prediction method for both, a general movement of the agents in the cartesian coordinate system and a circular movement in the cylindrical coordinate system. Furthermore, the basics of the event generation for the control of the agents are given.

Chapter 5 describes first the communication between the agents over an ideal network and second over an unreliable network. An overview over two common channel models from communication technology to represent the properties of a network is given. A Markov model of the wireless channel is presented, which is suitable for UAV applications.

Chapter 6 proposes the tasks of the event-based control units of the agents for an ideal communication network. The tasks of the control unit A_{S} of the stand-on agent and the tasks of the control unit A_{G} of the give-way agent are summarised.

Chapter 7 presents the parts of the control unit A_{G} of the give-way agent. The operating principles of the prediction unit G_{G}, the event generator E_{G} and the trajectory planning unit T_{G} are described. The event-based communication flow is given and a minimum inter event time span is derived. The method is summarised by an algorithm.

Chapter 8 describes the consequences on the event-based control method when information between the agents is transmitted over an unreliable network. The idea of handling the uncertainties arising from the network is motivated. Furthermore, the extended structure of the event-based control units A_S and A_G together with the delay estimators D_S and D_G is proposed.

Chapter 9 introduces the operating principle of the delay estimator and gives the extensions of the parts of the control units to cope with time delays and packet losses are given. The communication flow over an unreliable network is analysed and the method is summarised by an algorithm.

Chapter 10 introduces the quadrotor as the demonstration example. The quadrotor dynamics are stated and its control is derived. For the experiments the hardware of the physical quadrotor and the onboard software are introduced.

Chapter 11 presents the simulation results of three different scenarios. The analysis concentrates on three effects: time delays, packet losses and the possibility to fulfil the control aims even in the presence of packet losses.

Chapter 12 describes the experimental test bed and the results of the experiments with two quadrotors connected by an ideal communication network. In the scenario, the UAVs have to keep a formation while moving on circular trajectories. The analysis concentrates on the effects that are caused by the experimental environment. Furthermore, the event-based control method is applied to two robots in the same scenario to show that the control method is not limited to a single type of agents.

Chapter 13 summarises and concludes this thesis and states an outlook about possible future research directions.

Preliminaries 2

This chapter summarises the general notations used in this thesis in Section 2.1. A detailed list of symbols can be found in Appendix C. In Section 2.2 the notation of the position of an agent is introduced. The mathematical fundamentals for Bézier curves are stated in Section 2.3. In Section 2.4 the gradient descent optimisation method is presented. In Section 2.5 unmanned aerial vehicles (UAVs) and cars are presented, which serve as demonstration examples throughout this thesis

2.1 Notation

Throughout this thesis, scalars are represented by italic letters ($a \in \mathbb{R}$), vectors by bold italic letters ($\boldsymbol{x} \in \mathbb{R}^n$) and matrices by upper case bold italic letters ($\boldsymbol{A} \in \mathbb{R}^{n \times n}$). The entire set of real numbers is denoted by $\mathbb{R}$. $\boldsymbol{I}$ states the identity matrix of appropriate dimension. The i-th element of the j-th column of the matrix $\boldsymbol{A} \in \mathbb{R}^{n \times m}$ is given by $(\boldsymbol{A}_{ij})$ and the i-th entry of a vector $\boldsymbol{x}$ by $(x)_i$, respectively. $\mathbb{1} = \begin{pmatrix} 1 \dots 1 \end{pmatrix}^{\mathrm{T}}$ is the one vector and $\boldsymbol{0} = \begin{pmatrix} 0 \dots 0 \end{pmatrix}^{\mathrm{T}}$ the zero vector. The matrix $\boldsymbol{A} \in \mathbb{R}^{r \times c}$ with the entries a_{ij}, $i = 1, 2, \dots, r$ and $j = 1, 2, \dots, c$ is denoted by $\boldsymbol{A} = (a_{ij})$. The dimension of a vector $\boldsymbol{x}$ is denoted by $\dim(\boldsymbol{x})$ and $\mathrm{rank}(\boldsymbol{A})$ represents the rank of a matrix $\boldsymbol{A} \in \mathbb{R}^{n \times m}$. The transpose of a vector $\boldsymbol{x}$ or a matrix $\boldsymbol{A}$ is denoted by $\boldsymbol{x}^{\mathrm{T}}$ or $\boldsymbol{A}^{\mathrm{T}}$, respectively.

The i-th eigenvalue of $\boldsymbol{A} \in \mathbb{R}^{n \times n}$ is represented by $\lambda_i(\boldsymbol{A})$. The largest eigenvalue of $\boldsymbol{A} \in \mathbb{R}^{n \times n}$ is defined as

$$\lambda_{\max}(\boldsymbol{A}) = \max \{\lambda_i(\boldsymbol{A}),\ i = 1, 2, \dots, n\}$$

while

$$\lambda_{\min}(\boldsymbol{A}) = \min \{\lambda_i(\boldsymbol{A}),\ i = 1, 2, \dots, n\}$$

defines the smallest eigenvalue of $\boldsymbol{A} \in \mathbb{R}^{n \times n}$.

Sets are represented by calligraphic letters ($\mathcal{P}$), where their cardinality or size are denoted by $|\mathcal{P}|$. The absolute value of a scalar a is symbolised by $|a|$. $||\boldsymbol{x}||$ denotes the euclidean vector norm of $\boldsymbol{x} \in \mathbb{R}^n$ and $||\boldsymbol{A}||$ the compatible spectral norm of the matrix $\boldsymbol{A} \in \mathbb{R}^{n \times n}$, which are defined by

$$||\boldsymbol{x}|| = \sqrt{\sum_{i=1}^{N} (\boldsymbol{x})_i^2} \quad \text{and} \quad ||\boldsymbol{A}|| = \sqrt{\lambda_{\max}(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A})}, \tag{2.1}$$

respectively. Moreover, $||\boldsymbol{x}(t)||$ represents the euclidean vector norm at time instant t. $\mathrm{diag}(\lambda_i)$ is a diagonal matrix with the diagonal entries $\lambda_1, \lambda_2, \ldots, \lambda_N$.

A continuous-time linear time-invariant system is given by

$$\begin{aligned}\dot{\boldsymbol{x}}(t) &= \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) + \boldsymbol{E}\boldsymbol{d}(t), \quad \boldsymbol{x}(0) = \boldsymbol{x}_0 \\ \boldsymbol{y}(t) &= \boldsymbol{C}\boldsymbol{x}(t)\end{aligned}$$

where $\boldsymbol{x} \in \mathbb{R}^n$ denotes the state of the system with the initial value $\boldsymbol{x}_0$, the input $\boldsymbol{u} \in \mathbb{R}^m$ and the measured output $\boldsymbol{y} \in \mathbb{R}^r$. $\boldsymbol{d} \in \mathbb{R}^l$ represents exogenous disturbances. $\boldsymbol{A} \in \mathbb{R}^{n \times n}$, $\boldsymbol{B} \in \mathbb{R}^{n \times m}$, $\boldsymbol{E} \in \mathbb{R}^{n \times l}$ and $\boldsymbol{C} \in \mathbb{R}^{r \times n}$ are real matrices.

2.2 Specification of the position of an agent

Position of an agent in the cartesian coordinate system. In order to specify the position of an agent for a general movement the cartesian coordinate system is used. The position is denoted by

$$\boldsymbol{p}(t) = \begin{pmatrix} x(t) & y(t) & z(t) \end{pmatrix}^{\mathrm{T}}. \tag{2.2}$$

Position of an agent in the cylindrical coordinate system. For a circular movement of an agent the cylindrical coordinate system is used. The position (2.2) is transformed into cylindrical coordinates by

$$\begin{aligned} x(t) &= r\ \cos(\omega t) \\ y(t) &= r\ \sin(\omega t) \\ z(t) &= z(t). \end{aligned}$$

Angle between two agents moving on circular paths. In order to ease the calculation of the angle $\measuredangle(\boldsymbol{p}_1(t), \boldsymbol{p}_2(t))$ enclosed between two agents with positions $\boldsymbol{p}_1(t)$ and $\boldsymbol{p}_2(t)$, the phase $\Phi(\boldsymbol{p}(t))$ of the position $\boldsymbol{p}(t)$ of an agent given by (2.2) that moves on a circular path in the xy-plane is specified. For a circle with the centre $\boldsymbol{p}_\mathrm{m} = (x_\mathrm{m} \quad y_\mathrm{m} \quad z_\mathrm{m})^\mathrm{T}$ the phase $\Phi(\boldsymbol{p}(t))$ is defined as

$$\Phi(\boldsymbol{p}(t)) := \begin{cases} \arctan\left(\frac{y'(t)}{x'(t)}\right) & for\ x'(t) > 0,\ y'(t) \in \mathbb{R} \\ \arctan\left(\frac{y'(t)}{x'(t)}\right) + 180^\circ & for\ x'(t) < 0,\ y'(t) > 0 \\ \arctan\left(\frac{y'(t)}{x'(t)}\right) - 180^\circ & for\ x'(t) < 0,\ y'(t) < 0 \\ +90^\circ & for\ x'(t) = 0,\ y'(t) > 0 \\ -90^\circ & for\ x'(t) = 0,\ y'(t) < 0 \\ +180^\circ & for\ x'(t) < 0,\ y'(t) = 0 \end{cases} \tag{2.3}$$

with the relative position

$$\boldsymbol{p}'(t) = \begin{pmatrix} x'(t) \\ y'(t) \\ z'(t) \end{pmatrix} = \begin{pmatrix} x(t) - x_{\mathrm{m}} \\ y(t) - y_{\mathrm{m}} \\ z(t) - z_{\mathrm{m}} \end{pmatrix}$$

in relation to the centre of the circle. The phase of an agent corresponds to the angle of the cylindrical coordinates when the centre of the circle is at the origin of the coordinate system. Using the modulo operator defined as

$$\mathrm{mod}\ (a, m) = a - \left\lfloor \frac{a}{m} \right\rfloor \cdot m \tag{2.4}$$

the angle enclosed between two agents is determined as

$$\Phi_{\mathrm{diff}}(t) = \measuredangle(\boldsymbol{p}_1(t), \boldsymbol{p}_2(t)) = \mathrm{mod}\,(\Phi_1(t) - \Phi_2(t), 360°)$$

where $\Phi_1(t) = \Phi(\boldsymbol{p}_1(t))$ and $\Phi_2(t) = \Phi(\boldsymbol{p}_2(t))$ holds for simplicity and state the phases of the two agents, respectively.

2.3 Bézier curves

2.3.1 *Definition and properties*

Motivation. Numerically simple parametric curves are described using a parameter representation in the monomial base given in (2.15) on p. 36. However, in general the coefficients $\boldsymbol{a}_i$ of the curve do not provide much information about the course of the curve. Just $\boldsymbol{a}_0$ (start point of the curve) and $\boldsymbol{a}_1$ (tangential vector of the curve at $\boldsymbol{a}_0$) have concrete geometric interpretations. In contrast, the representation of the polynomials in the following Bernstein base allows to make predictions about the course of the curve on the basis of the coefficients. Hence, Bézier curves are used in this thesis for the trajectory planning of the agents.

Bernstein base on the unit interval $[0, 1]$**.** Bézier curves in the three-dimensional space of degree $m = n - 1$ are defined over the interval $u \in [0, 1]$ by their n control points $\boldsymbol{b}_k \in \mathbb{R}^3$ as [66]

$$\boldsymbol{v}(u) = \begin{pmatrix} v_{\mathrm{x}}(u) \\ v_{\mathrm{y}}(u) \\ v_{\mathrm{z}}(u) \end{pmatrix} = \sum_{k=0}^{m} \boldsymbol{b}_k \tilde{B}_k^m(u), \quad u \in [0, 1], \tag{2.5}$$

where $\tilde{B}_k^m(u)$, $(k = 0, 1, \ldots, m)$ are the Bernstein polynomials of order m given by

$$\tilde{B}_k^m(u) = \binom{m}{k} u^k (1 - u)^{m-k}, \ u \in [0, 1], \ k = 0, 1, \ldots, m. \tag{2.6}$$