Semi-Explicit MPC for Classes of Linear and Nonlinear Systems

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Chapter 1

Introduction

1.1 Motivation

Model predictive control (MPC) is a very successful modern control strategy which combines three striking advantages: First, constraints on system states and inputs can explicitly be incorporated. Second, a given performance criterion formulated regarding closed-loop behavior is directly optimized. Third, it is immediately applicable to nonlinear systems as well as to systems with multiple inputs. These advantages directly result from the general principle of model predictive control algorithms which allows to regard these aspects. In MPC typically an open-loop optimal control problem is solved online over a finite (prediction) time horizon using the current system state as initial condition. Only the first part of the optimal input sequence is applied to the plant and the procedure is repeated using a shifted time horizon and the updated system state as initial condition. This way, feedback is introduced. At the downside of MPC is the numerical effort needed to solve the optimal control problem during run time of the controlled system. In particular, the requirement to obtain a solution of the optimization problem sufficiently quickly with respect to the time-constants of the system to be controlled has largely restricted practical applications of MPC in the past.

Practical applications of MPC started to emerge in the late 1980s (and in single cases even earlier) in the process engineering industry. From these days on MPC has been applied for decades predominantly in this field. Prevalent therein are rather large but slow plants to be controlled. Typically only very few instances of each plant exist. Thus, dedicated and relatively powerful computational hardware could be utilized. As a result, the achieved computation times were fast enough to enable execution at the sampling rates required by these processes. Surveys on the history of MPC applications are found for example in (Lee, 2011; Mayne et al., 2000; Qin and Badgwell, 2003).

In the year 2003, the well cited survey paper on industrial MPC technology (Qin and Badgwell, 2003) still mainly reported traditional applications of MPC in the process engineering field. Yet, it was also recognized therein that "*Significant growth areas include the chemicals, pulp and paper, food processing, aerospace and automotive industries.*" Since then, this assessment has completely turned out to be true. In (Lee, 2011) we find that "*A significant number of applications involving mechanical and electronic systems are now being reported in the literature. [...] The reported applications include vehicle traction control, suspension, direct injection stratified charge engines, ducted fan in a thrust-vectored flight control experiment, automotive powertrains, magnetically actuated mass spring damper system, power converters, multicore thermal management, and so on.*" Further surveys as well as scanning the scope of today's application oriented publications in the area of MPC confirm this trend. Today, most novel applications for MPC are reported in automotive industry, see (Del Re et al., 2010; Di Cairano, 2012; Hrovat et al., 2012) and papers referenced therein. Various power electronic applications are reported and referenced in (Vazquez et al., 2014), aerospace applications are found e.g. in (Joos et al., 2012; Kang and Hedrick, 2009; Liu et al., 2011; Schlipf et al., 2013).

An increasing request for efficiency, for providing strict (safety) guarantees during operation and generally the demand to operate systems at high performance has been a driving force for the application of MPC also in these industries. MPC is perfectly suitable to satisfy these requests as has been acknowledged not only by theoretically oriented researchers but by practitioners as well. On the other hand, looking more in detail at the characteristics of such new applications, it is clear that they pose new requirements and challenges regarding the MPC algorithms to be employed. The controlled processes in novel fields are typically orders of magnitude faster than traditional ones. Consequently, the employed control algorithms have to keep up with that pace. At the same time, in contrast to classical applications of MPC, novel applications are typically produced in large volumes. This makes keeping the costs of the employed computational hardware to a minimum much more important than it used to be in classical applications and in many novel applications embedded computational hardware is to be employed. In turn, this means that fairly restricted resources in terms of computational power and memory are available for the execution of the control algorithms. This situation is alleviated to some extent by the fact that the plants in these applications can typically be modeled using few states and in many cases a linear system model is sufficient. Summarizing, in order to be able to employ MPC in novel applications and exploit its advantages in these fields, fast and efficient MPC algorithms are required which can be executed at high sampling rates even on low cost and embedded hardware.

Recent and ongoing research addresses this task from several directions and also the thesis at hand contributes towards this goal. Among the existing results, firstly tailored numerical optimization schemes are developed, see for example (Diehl et al., 2009; Domahidi et al., 2012; Wang and Boyd, 2010). Second, more attention is paid to the interplay of optimization algorithms with the (embedded) computational hardware they are run on. This is, among others, done in (Jerez et al., 2014; Zometa et al., 2012) and in the papers referenced in (Jones and Kerrigan, 2015) and in (Lucia et al., 2016). A third strategy addresses shifting some numerical effort offline in order to alleviate online computations. The most prominent representative of this class of approaches is so-called explicit MPC where, in advance, an explicit solution of the optimal control problem is pre-computed offline as a function of the state. This function is stored and made available online where it only has to be evaluated for the current system state, see for example (Alessio and Bemporad, 2009; Bemporad et al., 2002; Tøndel et al., 2003) for results regarding linear systems and (Darup and Mönnigmann, 2012; Johansen, 2004; Summers et al., 2010) for nonlinear systems. At the core of this approach is the idea to consider the finite horizon open-loop optimal control problem underlying MPC as a multi-parametric program, an optimization problem which depends on a parameter, here the system state. Despite some advantages explicit MPC definitely has, the approach is restricted to rather small MPC problems. For growing problem dimensions, the structure of the explicit solution quickly grows too complex to evaluate it efficiently online. Beyond that, even when considering small MPC problems, it is by no means clear that the most efficient online algorithm is obtained from shifting either all or none of the optimization offline. Intermediate strategies might be superior. In addition to that, in high volume applications it is especially attractive to invest a higher offline effort to simplify online computations and enable the employment of cheaper computational hardware. In this case, a higher one-time (offline) invest pays back with the savings achieved in each instance of the controlled system sold.

Thus, intermediate “semi-explicit” MPC strategies, which combine the so-far mainly separate worlds of explicit and online optimization based MPC algorithms, are an obvious and at the same time very promising extension to the available collection of MPC schemes. They have the potential to satisfy the application driven current demand for fast and efficient MPC algorithms. With respect to completely explicit approaches, clearly improved scalability in terms of the problem size can be expected; with respect to purely online optimization based schemes, a simplified numerical optimization can be expected. Thus, semi-explicit approaches can extend applicability of MPC to system classes beyond what is currently possible or beyond where the application of MPC is currently profitable. In the survey on explicit MPC (Alessio and Bemporad, 2009), this assessment is shared: *“Future research efforts should therefore pursue three main directions. [...] Third, semi-explicit methods should be also sought, in order to pre-process of line as much as possible of the MPC optimization problem without characterizing all possible optimization outcomes, but rather leaving some optimization operations on-line.”* Additionally, such combination is clearly interesting from a conceptual point of view.

In this thesis, we aim at presenting a semi-explicit MPC approach which fulfills these expectations: An MPC approach is introduced which joins concepts of the explicit and of the online optimization based approach to combine their individual strengths. The resulting MPC scheme should be quickly executable and require only little computational resources. This way, the results of this thesis shall contribute to the applicability of MPC to new problem classes. At the same time, above discussed benefits of MPC should be maintained by the semi-explicit MPC scheme and system theoretic properties such as feasibility of the optimization, high control performance and asymptotic stability of the closed loop should be guaranteed.

1.2 Contributions and outline of the thesis

The basic principle of the semi-explicit approach contributed in this thesis is as follows: For a given specific MPC problem, in a first offline stage state-dependent parametrizations are computed such that they optimally approximate solutions to the MPC optimization problem. In the second online phase, these parametrizations are employed to simplify the numerical solution of the optimization. Based thereon, a simple and efficient overall online MPC scheme is formulated. A main characteristic of the proposed approach is that the parametrizations are computed data-based, namely via the application of a suitably adapted subspace clustering algorithm which is applied to optimal input sequences. Throughout the thesis this general approach will be elaborated for different problem classes so that overall the approach is applicable to linear as well as to several classes of nonlinear systems.

In the following, the contributions and the structure of this thesis is outlined more in detail.

Chapter 2

Background on MPC and parametric optimization

In this chapter the basic problem setup as well as basic theoretical results underlying the material presented in this thesis are introduced. First, we review in the area of model predictive control some basic theoretic results. Second, we have a closer look at parametric optimization problems as they are at the core of the considered class of MPC problems and as the theory presented in this work essentially addresses efficiently solving them for fixed parameter taken from a given set of parameters.

2.1 Model Predictive Control

We consider control of nonlinear time-invariant discrete-time systems of the form

$$x_{k+1} = f(x_k, u_k),\ k \geq 0, \tag{2.1}$$

with x_0 given, where $f : \mathbb{X} \times \mathbb{U} \to \mathbb{R}^n$ holds, $x_k \in \mathbb{X} \subseteq \mathbb{R}^n$ is the system state and $u_k \in \mathbb{U} \subset \mathbb{R}^m$ is the control input at time $k \in \mathbb{N}$. The sets $\mathbb{X}$ and $\mathbb{U}$ define state and input constraints, respectively. We assume that the system has an equilibrium $\overline{x}$ in the interior of $\mathbb{X}$, that is $\overline{x} \in \mathrm{int}(\mathbb{X})$, with corresponding input $\overline{u} \in \mathbb{U}$ such that $\overline{x} = f(\overline{x}, \overline{u})$ holds. To simplify matters we assume throughout the thesis without loss of generality that the equilibrium is at the origin, that is $\overline{x} = 0$ and $\overline{u} = 0$.

MPC for setpoint regulation

Model predictive control in a basic form addresses stabilization of a setpoint $\overline{x}$, $\overline{u}$ of a system (2.1) subject to the minimization of a given performance criterion. The performance objective is expressed via a cost function $\ell : \mathbb{X} \times \mathbb{U} \to \mathbb{R}_{\geq 0}$ which is positive definite with respect to the setpoint $\overline{x}$, $\overline{u}$ and whose sum along closed-loop trajectories is to be minimized. At the core of such basic MPC scheme is the following finite horizon open-loop optimal control problem

$$\begin{aligned} V^*(x_k) = \min_{U_k, X_k}\ & J_s(X_k, U_k) \\ \text{s.t. }\ & x_{j+1|k} = f(x_{j|k}, u_{j|k}),\ j = 0, \dots, N-1 \\ & x_{j|k} \in \mathbb{X},\ j = 1, \dots, N-1 \\ & u_{j|k} \in \mathbb{U},\ j = 0, \dots, N-1 \\ & x_{N|k} \in \mathbb{X}_T \\ & x_{0|k} = x_k \end{aligned} \tag{2.2}$$

where

$$J_s(X_k, U_k) = \sum_{j=0}^{N-1} \ell(x_{j|k}, u_{j|k}) + V_T(x_{N|k}) \tag{2.3}$$

holds. Therein, $U_k = [u_{0|k}^\top, \ldots, u_{N-1|k}^\top]^\top \in \mathbb{R}^{mN}$ denotes a stacked predicted input sequence at time k over a prediction horizon of $N \in \mathbb{N}$ steps. The corresponding stacked state sequence is denoted by $X_k = [x_{1|k}^\top, \ldots, x_{N|k}^\top]^\top \in \mathbb{R}^{nN}$. Furthermore, $\mathbb{X}_T \subseteq \mathbb{X}$ and $V_T : \mathbb{X}_T \to \mathbb{R}_{\geq 0}$ are a terminal constraint set and a terminal cost function, respectively.

In order to ensure existence of a minimum of Optimization (2.2), further assumptions are required. Different sets of assumptions are sufficient to guarantee this existence, of which we use the following rather intuitive ones as standing assumptions throughout this thesis.

Assumption 2.1. *Let the functions f, ℓ and V_T be continuous. Let the sets $\mathbb{X}$ and $\mathbb{X}_T$ be closed and let the set $\mathbb{U}$ be compact.*

Without further assumptions, Optimization (2.2) clearly might have a non-unique solution. Whenever this is relevant, we assume that a unique choice among the solutions is made.[1] By $U_k^*(x_k) = [{u_{0|k}^*}^\top(x_k), \ldots, {u_{N-1|k}^*}^\top(x_k)]^\top$ we denote the minimizing input sequence of Optimization (2.2). The standard MPC algorithm based on this optimization is then as follows.

Algorithm 1 Basic MPC algorithm

1: obtain current state x_k
2: solve Optimization (2.2)
3: apply $u = u_{0|k}^*(x_k)$ to the plant and return to step 1

Application of this algorithm results in the closed-loop system

$$x_{k+1} = f(x_k, u_{0|k}^*(x_k)). \tag{2.4}$$

Considering that the goal is stabilization of the origin, relevant questions concerning this scheme are: i) If the optimization is feasible at time k for the state x_k, is it also feasible at time $k+1$ for the closed-loop state x_{k+1}? This property is called *recursive feasibility*. ii) Is the origin an asymptotically stable equilibrium of the closed-loop system (2.4)? iii) If so, what is the region of attraction of the origin? In order to be able to give satisfactory answers to these questions, some further assumptions regarding the setup are required.

Assumption 2.2. *Let ℓ be positive definite in both of its arguments.*

Assumption 2.3 (Mayne et al. (2000))**.** *Assume that the terminal constraint set $\mathbb{X}_T$, the terminal control law κ and the terminal cost function V_T are such that the following holds:*

1. *The terminal constraint set $\mathbb{X}_T$ fulfills $\mathbb{X}_T \subseteq \mathbb{X}$, it is closed and $0 \in \mathbb{X}_T$.*
2. *There is a terminal control law $\kappa : \mathbb{X}_T \to \mathbb{R}^m$, $x \mapsto \kappa(x)$ such that $\kappa(x) \in \mathbb{U}$ for all $x \in \mathbb{X}_T$.*

[1] For the results in this thesis, we implicitly rely on some regular structure of the optimizer of (2.2) as a function of the state x. If the minimizer is unique, such regular structure is given. In non-unique cases, we assume that the selection function used to obtain a unique solution preserves this structure.

3. *The terminal constraint set is invariant under the terminal control law, i.e., for all* $x \in \mathbb{X}_T$ *it holds that* $f(x, \kappa(x)) \in \mathbb{X}_T$.

4. *The terminal cost function* V_T *satisfies* $V_T(x) - V_T(f(x, \kappa(x))) \geq \ell(x, \kappa(x))$ *for all* $x \in \mathbb{X}_T$ *and* $V_T(0) = 0$.

The following result holds.

Theorem 2.1 (Mayne et al. (2000))**.** *Let Assumptions 2.2 and 2.3 hold. Algorithm 1 is recursively feasible, the origin is an asymptotically stable equilibrium of the closed-loop system* (2.4)*, the region of attraction is the set of states for which Optimization* (2.2) *is feasible.*

The main idea for the proof of the latter theorem is as follows. Due to the assumptions on the stage cost function ℓ, the optimal value function V^* fulfills the properties of a Lyapunov candidate function. If the optimization has been solved before, in each time step a candidate solution U_C to the optimization exists consisting of the shifted previous predicted input sequence with the input obtained via the terminal control law appended. The cost of this candidate solution is an upper bound on the cost of the optimal solution. Assumptions 2.2 and 2.3 ensure that a certain cost decrease of the candidate solution is achieved, which implies that the optimal value function V^* fulfills the properties of a Lyapunov function. In fact results are available which make use of a weaker version of Assumption 2.2. For example, positive definiteness of ℓ with respect to an output function together with a corresponding detectability condition are sufficient to prove asymptotic stability of the loop closed with the above scheme. More information on the computation of a suitable terminal cost and terminal constraint set can be found in (Chen and Allgöwer, 1998) for the continuous time case and in (Rawlings and Mayne, 2009) for the discrete-time version.

Under some slightly stronger assumptions on the MPC ingredients, the requirements for Algorithm 1 can be relaxed such that not necessarily an optimal solution is required therein but the results remain valid if initially any feasible input sequence is chosen which is then improved along closed-loop trajectories, c.f. (Scokaert et al., 1999).

Further approaches to guarantee closed-loop stability

Besides the discussed approach to achieve and prove stability of the closed-loop system, a variety of different concepts exists for this purpose, see the overview in (Mayne et al., 2000). Dual mode schemes basically use an MPC algorithm as introduced above to drive the system state into a region around the origin and therein switch to a local control law exploiting its stabilizing properties. Variable horizon schemes reduce the prediction horizon length during operation, thereby automatically ensuring a decrease of the value function. Contractive and stability enforced MPC schemes ensure closed-loop stability employing in some way an additional control Lyapunov function which has to be known explicitly a priori. Unconstrained MPC schemes do not employ any terminal constraints and, nevertheless, allow to formulate closed-loop stability guarantees for sufficiently long prediction horizons, typically under additional assumptions on the system to be controlled, see (Grüne, 2012) for an overview.

Basically all of these schemes have at their core an optimization problem which depends on and online has to be solved for the current system state. These problems are structurally